

Particle Count Reduction in an E_8 Standard Model

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Abstract

By definition in Lie Algebras, all roots can be composed from either all positive or all negative combinations of their "simple roots". Taking a modified A.G. Lisi split real even E_8 model with 240 fundamental physics particles associated with an extended Standard Model, a particle count reduction (from 240 fundamental particles to 8 "elemental" particles) is determined from these 8 simple roots. Interestingly, by taking account of the particle mass assignments, all known fermions $\{e/\nu, u/d, c/s, t/b\}$, as well as known (plus the Lisi predicted) bosons $\{W/B, \text{gluons}(g), \omega, e\phi, x\Phi\}$ can be generated with the sum of the simple root masses being less than the resulting composite particle masses (with the exception of the four 2nd and 3rd generation leptons $\{e_{\mu,\tau} / \nu_{\mu,\tau}\}$).

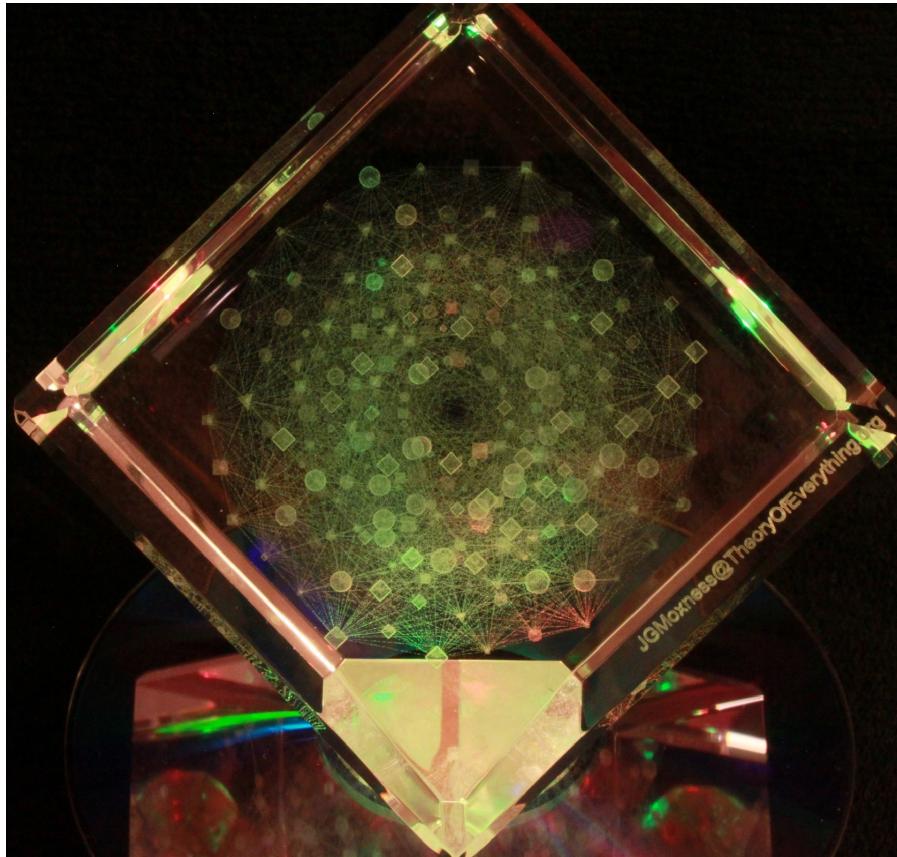


Figure 1: 2D photo of an 8D→3D orthographic projection of E_8 . It is laser etched into a 3"x3"x3" crystal (perspective) 3D object, with vertex shape and size assigned based on extended Standard Model particle assignments

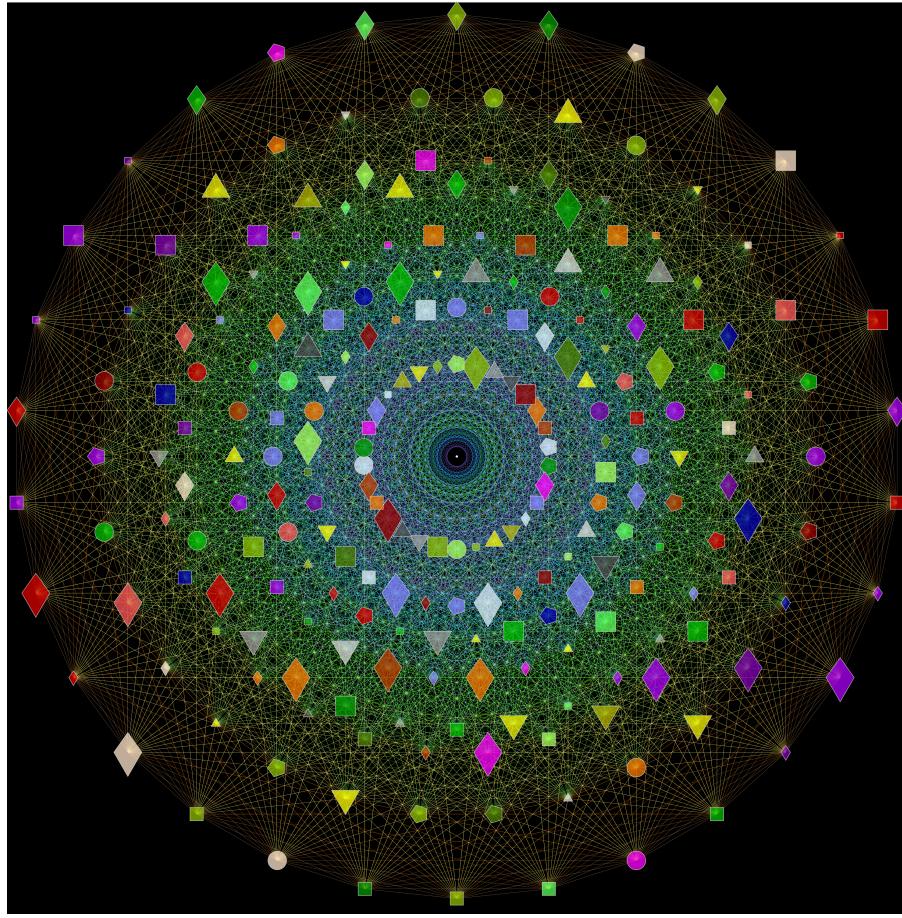


Figure 2: 8D-to-2D Petrie projection of E_8 (a.k.a the 4_{21} polytope's Gosset figure), the orthographic "shadow" of the 3D object in Figure 1, with 6720 shortest edges of 8D norm'd length $\sqrt{2}$ and 240 vertices with shape, size, color/shade assigned based on extended Standard Model particle assignments

Introduction

Visualizing the 8 dimensional (8D) E_8 polytope uses 2 (or 3) basis vectors to project it into 2D (or 3D). Figures 1, 2, and 22 (Appendix D) are generated using an 8D orthographic projection operation (1) applied to the split real even (SRE) E_8 vertices with basis vectors (2), (3) and (4). As Figure 1 is in crystal as a (perspective) 3D object, looking closely at it creates a parallax effect. From a distance, as in the photo, the (orthographic) "shadow" of this polytope can be seen as the Petrie projection (Figure 2), also known as the 4_{21} polytope's Gosset figure. All visualizations in this paper were created with the VisibLie_Dynkin _& E8 *Mathematica*TM notebook. The user interfaces for these are shown in Appendix C.

$$\text{VertexProjectionLocation}_i = \{H.\text{SRE}_{i=1-256}, V.\text{SRE}_{i=1-256}, Z.\text{SRE}_{i=1-256}\} \quad (1)$$

$$H = \left\{ \frac{1}{4} \sqrt{\frac{2}{5 - \sqrt{5} + \sqrt[3]{(5 + \sqrt{5})^2}}} , \frac{1}{\sqrt{\sqrt{30 - 6\sqrt{5}} + 3(5 + \sqrt{5})}} , \frac{1}{\sqrt{\sqrt{30 - 6\sqrt{5}} + 3(5 + \sqrt{5})}} , \frac{2}{\sqrt{15 + 6\sqrt{5} + \sqrt{3(85 + 38\sqrt{5})}}} \sin\left[\frac{2\pi}{15}\right] , \frac{2\cos\left[\frac{\pi}{15}\right]}{\sqrt{\sqrt{30 - 6\sqrt{5}} + 3(5 + \sqrt{5})}} , 0, \frac{2\cos\left[\frac{\pi}{15}\right]}{\sqrt{\sqrt{30 - 6\sqrt{5}} + 3(5 + \sqrt{5})}} \right\} = \quad (2)$$

$[0, -0.556793440452, 0.19694925177, -0.19694925177, 0.0805477263944, -0.385290876171, 0., 0.385290876171]$

$$V = \left\{ \frac{1}{4} \sqrt{\frac{1}{7 - 3\sqrt{5} + \sqrt{\frac{1}{15}(50 - 22\sqrt{5})}}} , 0, \frac{1}{\sqrt{\frac{1}{30}(15 + \sqrt{75 - 30\sqrt{5}})}} \sin\left[\frac{\pi}{15}\right] , \frac{1}{\sqrt{\frac{1}{30}(15 + \sqrt{75 - 30\sqrt{5}})}} \sin\left[\frac{\pi}{15}\right] , 0, \frac{\frac{(5 - \sqrt{5})(15 + \sqrt{75 - 30\sqrt{5}})}{30(5 + \sqrt{5})}}{\sin\left[\frac{\pi}{15}\right]} \right\} \sin\left[\frac{\pi}{15}\right] = \quad (3)$$

$$\left| V \right| = [0.180913155536, 0., 0.160212955043, 0., 0.0990170516545, 0.766360424875, 0.0990170516545]$$

$$Z = \left\{ 0, -\frac{2}{\sqrt{15 + 6\sqrt{5} + \sqrt{3(85 + 38\sqrt{5})}}} \sin\left[\frac{2\pi}{15}\right] , -\frac{1}{\sqrt{\sqrt{30 - 6\sqrt{5}} + 3(5 + \sqrt{5})}} , -\frac{1}{\sqrt{\sqrt{30 - 6\sqrt{5}} + 3(5 + \sqrt{5})}} , \frac{2}{\sqrt{15 + 6\sqrt{5} + \sqrt{3(85 + 38\sqrt{5})}}} \cos\left[\frac{\pi}{15}\right] , 0, -\frac{2\cos\left[\frac{\pi}{15}\right]}{\sqrt{\sqrt{30 - 6\sqrt{5}} + 3(5 + \sqrt{5})}} \right\} = \quad (4)$$

$[0, -0.0805477263944, -0.19694925177, 0.19694925177, 0.556793440452, 0.385290876171, 0, -0.385290876171]$

Lisi has proposed an extended Standard Model (SM) based on a SRE E_8 Lie Algebra with a fundamental physics particle associated with each of its 240 roots [1]. In this model, particle assignments are modified slightly in order to create a pattern of roots consistent with its Simple Roots (SRs). This construction of the SRE E_8 is based on the $256 = 2^8$ binary pattern from the 9th row of the Pascal Triangle {1, 8, 28, 56, 70, 56, 28, 8, 1} and its associated Cl(8) Clifford Algebra. The SRE E_8 roots are defined by combining the 112={56, 56} integer roots of Lie group $D_8=\text{SO}(16)$ and the 128={1, 28, 70, 28, 1} half integer roots of Lie group $C_8=\text{Sp}(16)$. Specifically, C_8 contains all permutations of $\{\pm 1, \pm 1\}/2$ with an even number of plus signs (an 8-Demi-Cube or even 7-Cube), which are assigned to the 2nd and 3rd generation fermions. D_8 contains all permutations of $\{\pm 1, \pm 1, 0, 0, 0, 0, 0, 0\}$, which are assigned to 48 bosons and the 64 1st generation fermions. In this model, the 16 particles associated with {8, 8} are excluded as dimensional generators from the permutations of $\{\pm 1, 0, 0, 0, 0, 0, 0, 0\}$. These excluded particles are associated with the 8 Orthoplex (dual of the 8-Cube with 256 vertices). E_8 has 120 positive roots and 120 negative roots. These construction patterns are shown in Figure 3.

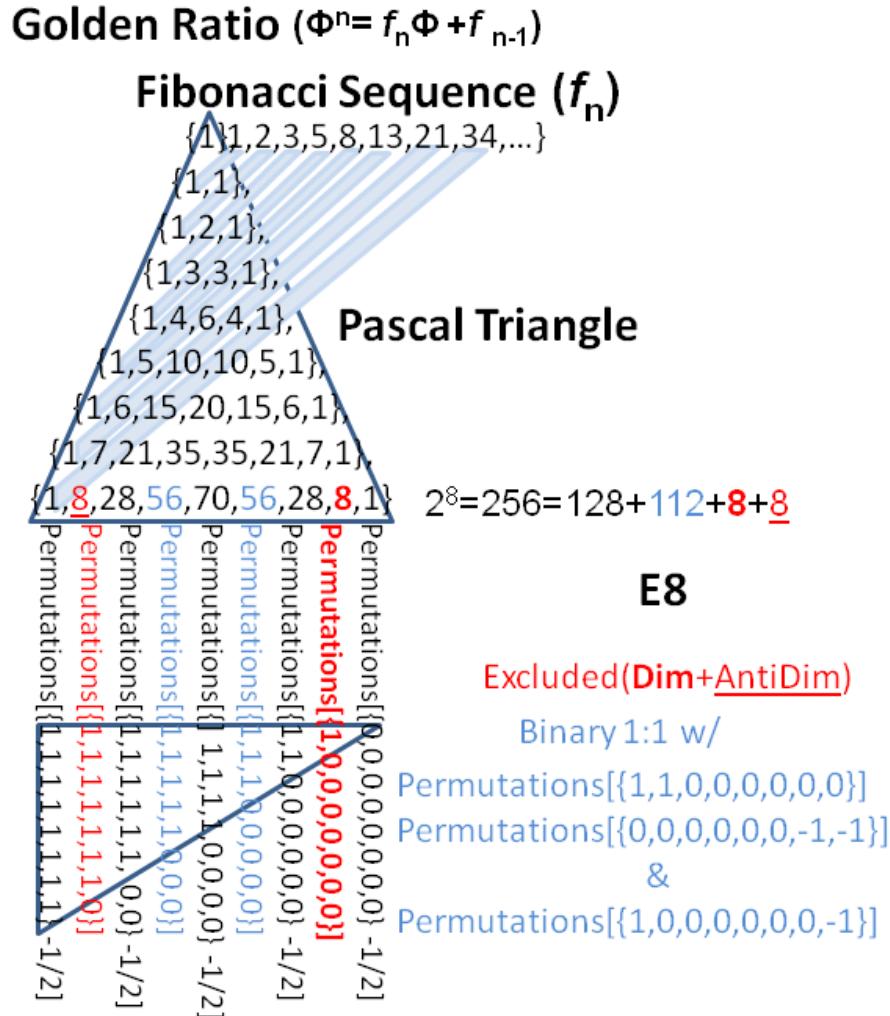


Figure 3: SRE E_8 construction from Pascal Triangle, Cl(8) Clifford Algebra and binary permutations

The linkage of Lie algebra E_8 to the Pascal triangle is shown in Figure 4 with the basis vectors (5) used for 8D \rightarrow 2D orthographic projection of the "Binary" coordinates (the 2 valued {0, 1} vertex dimensions, as opposed to the 5 valued {0, ± 1 , $\pm 1/2$ } dimensions of SRE). This binary representation is a Little Endian (right most significant) zero-based 8 dimensional vector {0-7}. Notice the number of nodes in each column are in 1:1 correspondence with the 9th row of the Pascal triangle (after accounting for the overlaps (6), of course). These are projected from left to right and bottom to top isomorphically with the lexicographical sort of Appendix B. Also notice that the linear basis vectors. Similar projections using the SRE coordinates of the C_8 7-Cube and 4-Cube Tesseract as sub-groups within E_8 are also available within VisibLie_Dynkin _&_E8. As in most of these figures, the edge line coloring is varied based on projected distance from the origin of the outer most vertex. Figure 4 has 1024 unit length edges.

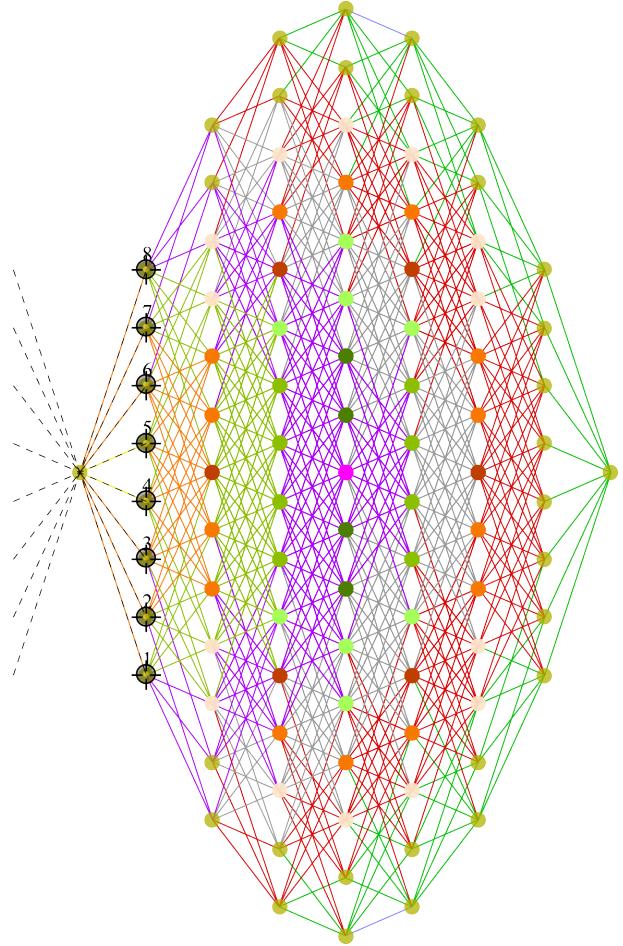


Figure 4: 8D \rightarrow 2D orthographic projection of binary coordinates projected using basis vectors (5) with overlaps (6). The number of vertices in each column represents the 9th row of the Pascal triangle {1, 8, 28, 56, 70, 56, 28, 8, 1}.

Note: If viewing *Mathematica*TM Computable Document Format (.CDF), this figure is interactive with vertex detail information on mouse-over.

$$H = \left\{ \frac{1}{4\sqrt{2}}, \frac{1}{4\sqrt{2}}, \frac{1}{4\sqrt{2}}, \frac{1}{4\sqrt{2}}, \frac{1}{4\sqrt{2}}, \frac{1}{4\sqrt{2}}, \frac{1}{4\sqrt{2}}, \frac{1}{4\sqrt{2}} \right\}$$

$$V = \left\{ -\frac{\sqrt{\frac{7}{6}}}{2}, -\frac{5}{2\sqrt{42}}, -\frac{\sqrt{\frac{3}{14}}}{2}, -\frac{1}{2\sqrt{42}}, \frac{1}{2\sqrt{42}}, \frac{\sqrt{\frac{3}{14}}}{2}, \frac{5}{2\sqrt{42}}, \frac{\sqrt{\frac{7}{6}}}{2} \right\} \quad (5)$$

$$\text{InView vertices} = \{\{\text{overlap}, \text{count}\}, \dots\} \text{ Total}$$

$$\{\{1, 38\}, \{2, 14\}, \{3, 14\}, \{4, 6\}, \{5, 8\}, \{6, 8\}, \{7, 4\}, \{8, 1\}\} 93 \quad (6)$$

$$\text{All vertices} = \{\{\text{overlap}, \text{count}\}, \dots\} \text{ Total}$$

$$\{\{1, 38\}, \{2, 28\}, \{3, 42\}, \{4, 24\}, \{5, 40\}, \{6, 48\}, \{7, 28\}, \{8, 8\}\} 256$$

The E_8 Dynkin diagram (Figure 5) with canonical node ordering generates the Cartan matrix (CM) in (7), which is used to construct the E_8 Lie Algebra.

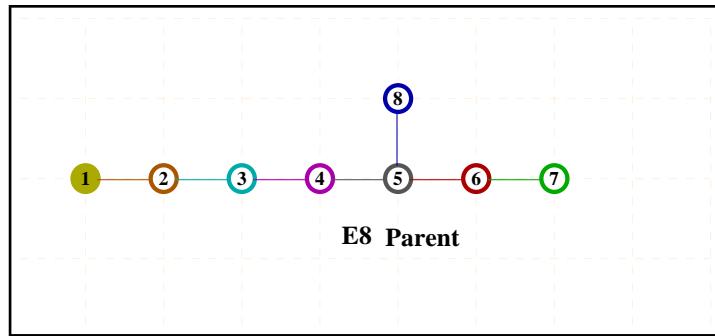


Figure 5: E_8 Dynkin with canonical node ordering

$$CM = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 2 \end{pmatrix}, SRM = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \quad (7)$$

By definition in Lie Algebras, all roots can be composed from either all positive or all negative combinations of the SRs. The simple root matrix (SRM) in (7) and the 256=240+16 excluded SRE vertices are used to uniquely generate all 240=120 positive+120 negative algebra roots of E_8 using (8), ignoring the excluded 16=8 positive (dimensional)+8 negative (anti-dimensional) SRE generated roots. The full list of these roots are shown in Appendix B and C.

$$\text{AlgebraRoot}_i = [SRM^T]^{-1} \cdot \text{SRE}_{i=1-256} \quad (8)$$

New Model Construction

In this new model, each E_8 vertex is an 8 dimensional vector that contains the configuration of a particle's spin in positions {1, 2, 3, 4}, the generations in position {5}, and color in positions {6, 7, 8}.

As is well known in the Lisi SRE model, there are only 48 assigned D_8 integer bosons and only 128 C_8 half-integer vertices available. Yet, with 192=64x3 generation fermions in SM, the meaning or validity of assigning a generation of fermions to the remaining 64 D_8 integer vertices has been hotly debated [2]. In this model these remaining integer fermions are assigned to the 1st generation. This means that the integer SRE vertices are fully allocated with the "generation 0" bosons and 1st generation fermions. For a complete reference of particle assignments, see Appendix B.

The 1:1 bit-wise correspondence of a particle's quantum number assignments, a Big Endian (left most significant) zero-based 8 dimensional vector {7-0}, are respectively {1 bit=a (Antiparticle- p/\bar{p}), 1 bit=p (Type- e/v or u/d quark), 2 bits=c1, c0 (Color- r/g/b/none), 2 bits=s1, s0 (Spin- \hat{L} , \hat{R} , $\hat{\bar{L}}$, $\hat{\bar{R}}$), 2 bits=g1, g0 (Generations- $3\tau/2\mu$ half integer fermions, 1e/0 integer bosons)} or simply {a, p, s1, s0, c1, c0, g1, g0}. The green bold type face indicates quantum assignments which are not exclusively allocated to a dimension {7-0} defined in this model, but in the particle assignments based on the inherent structural symmetry of E_8 . These construction patterns are shown in Figure 6 and 7.

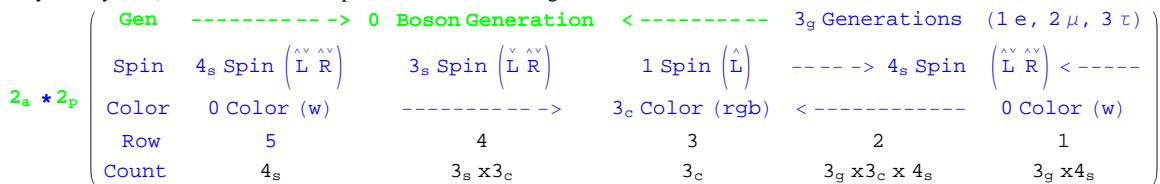


Figure 6: Particle flavor counts given quantum number assignments

2_a	$\begin{array}{ccccccc} \text{Gen} & \xrightarrow{\text{-----}} & \text{0 Boson Generation} & \xleftarrow{\text{-----}} & \text{3}_g \text{ Generations } (1_e, 2_\mu, 3_\tau) \\ \text{Spin} & 4_s \text{ Spin } (\hat{L} \hat{R}) & 3_s \text{ Spin } (\overset{\vee}{L} \overset{\vee}{R}) & 1 \text{ Spin } (\hat{L}) & \xrightarrow{\text{----}} 4_s \text{ Spin } (\hat{L} \hat{R}) & \xleftarrow{\text{----}} 0 \text{ Color } (w) \\ \text{Color} & 0 \text{ Color } (w) & & 3_c \text{ Color } (rgb) & & & \end{array}$
	$\begin{array}{ccccccc} \text{Row} & 5 & & 4 & & 3 & 2 & 1 \\ \text{p = 1} & \text{Ex}_{5-8} & \{ \{ \{ e_s \phi, e_t \phi \}, \text{B} \} & \{ \{ \{ \omega_L, \omega_R \}, \text{W} \} \} & \{ u, c, t \} & \{ v_e, v_\mu, v_\tau \} \\ \text{p = 0} & \text{Ex}_{1-4} & \{ \{ x_1 \bar{x}, x_2 \bar{x}, x_3 \bar{x} \} & \{ g^{ab}, g^{r\bar{b}}, g^{r\bar{a}} \} \} & \{ d, s, b \} & \{ e, e_\mu, e_\tau \} \end{array}$

Figure 7: Particle flavors in row / column groups with boson (group) coloring based on Lie group assignments ($F4, F4^S, D4 & G2, G2^S$)

The anti-particle **{a}** bit is associated with the negation of a particle vertex coordinate. This creates a given particle's anti-particle. It is helpful to note that the entire binary and SRE vertex list is lexicographically ordered from negative to positive with a perfect mirroring about the middle, between the 128th and 129th of 256 vertices, which are the \hat{R} muon neutrinos (ν_μ and $\bar{\nu}_\mu$). Also of interest, the first and last particles in the list are the \hat{R} tau neutrinos (ν_τ and $\bar{\nu}_\tau$). This aligns well with the idea that it is associated with (T)ime reversal in the CPT conservation laws. While the E_8 algebra roots in the SRE ordered list are not lexicographically ordered, it does exhibit the same mirrored pattern of positive (negative) roots as do the binary and SRE particle (anti-particle) assignments.

The **{g0}** bit splits the generation 0 boson family of 128 integer roots (and integer Spins) of D_8 and 8-Orthoplex from the 128 half integer root (and half integer spin) of C_8 fermions.

The **{p}** bit splits the particle families into two types, referenced in the leptons as electron and neutrino types, while the quarks are designated by up and down types. The differences are most easily seen in the 8x8 rotation matrix used for transforming SRE coordinates to physics coordinates (9) and those matrices used in identifying bosonic (10) and fermionic (11) triality transformations. These matrices are divided by an upper left quadrant affecting the SRE {1-4} spin positions and a lower right quadrant affecting the SRE {5-8} generation-color positions.

$$\text{Physics_Rotation} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} \end{pmatrix} \quad (9)$$

$$\text{Bosonic_Triality} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (10)$$

$$\text{Fermionic_Triality} = \begin{pmatrix} 1 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 2 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 2 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{pmatrix} \quad (11)$$

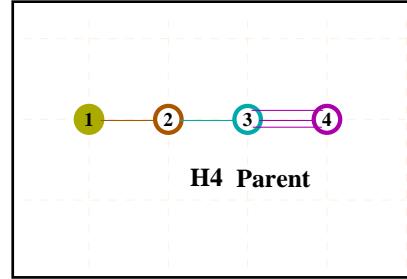


Figure 8: H_4 Dinkin diagram

This left-right splitting of E_8 may be related to the idea that the Dynkin diagrams can be folded, generating related sub-groups. The E_8 Dynkin folds into H_4 (Figure 8). It is associated with the 600-Cell, a 4D polytope (or polychora) of 120 vertices (Figures 9 and 10). It has a dual, the 120-Cell of 600 vertices (Figures 11 and 12). This 600-Cell is constructed from the combination of the 96 vertices of the snub 24-Cell and the 24 vertices of the 24-Cell (Figure 13 and 14), which is self-dual with the most symmetrical Dynkin diagram D_4 (Figure 15). It is interesting to note that the 24-Cell is constructed from the 16 vertices of the Tesseract (or 8-Cell or 4-Cube as shown in Figure 16) and the 8 vertices of its dual, the 4-Orthoplex (or 16-Cell). All of these polychora can be found within E_8 with the excluded 8-Orthoplex. The snub 24-Cell is constructed from even permutations of $\{\phi, 1, 1/\phi, 0\}$, where $\phi = \frac{1}{2}(\sqrt{5} + 1)$ is the golden ratio with numerical value of 1.61803... and thus can not be found directly within E_8 .

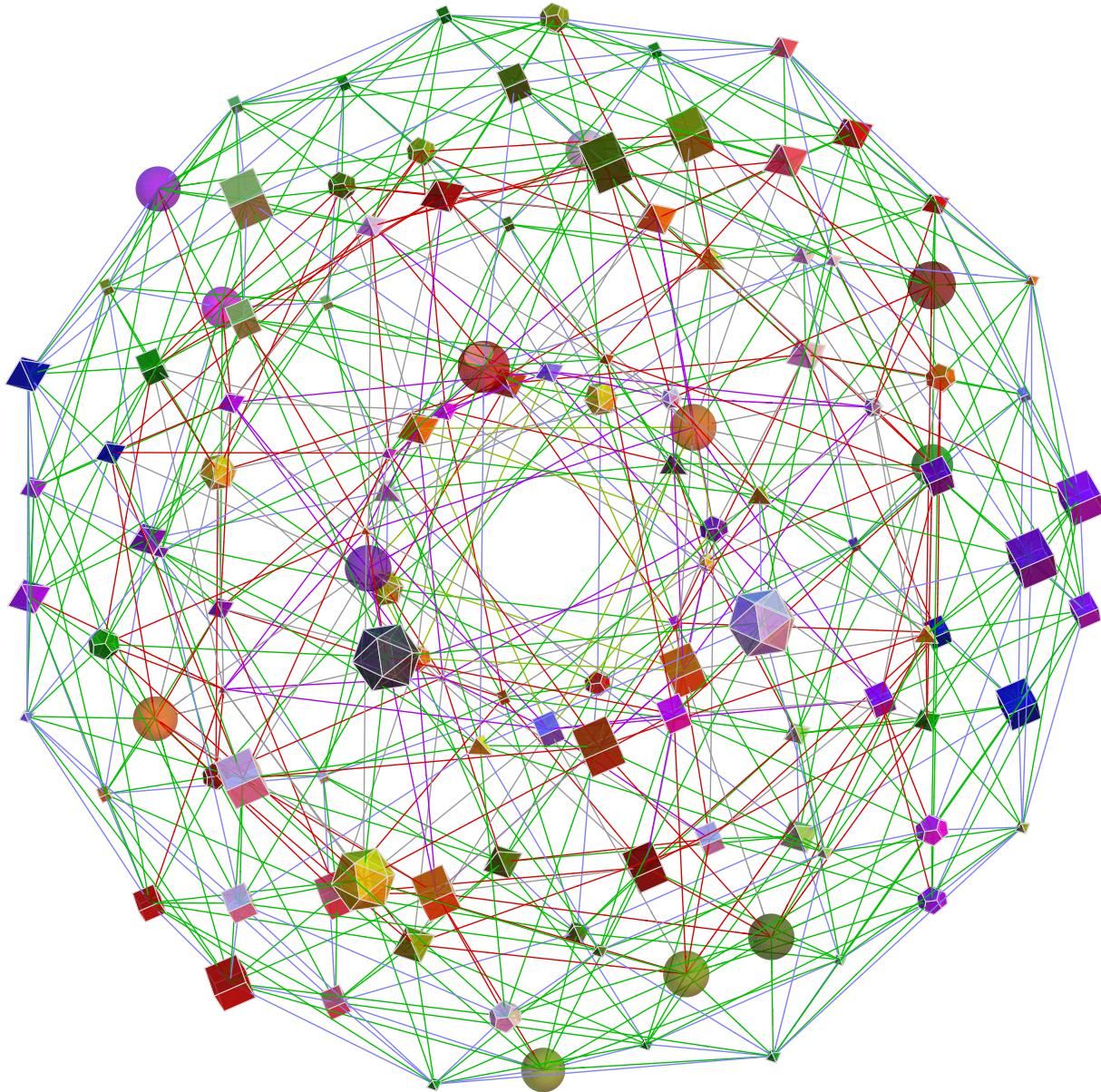
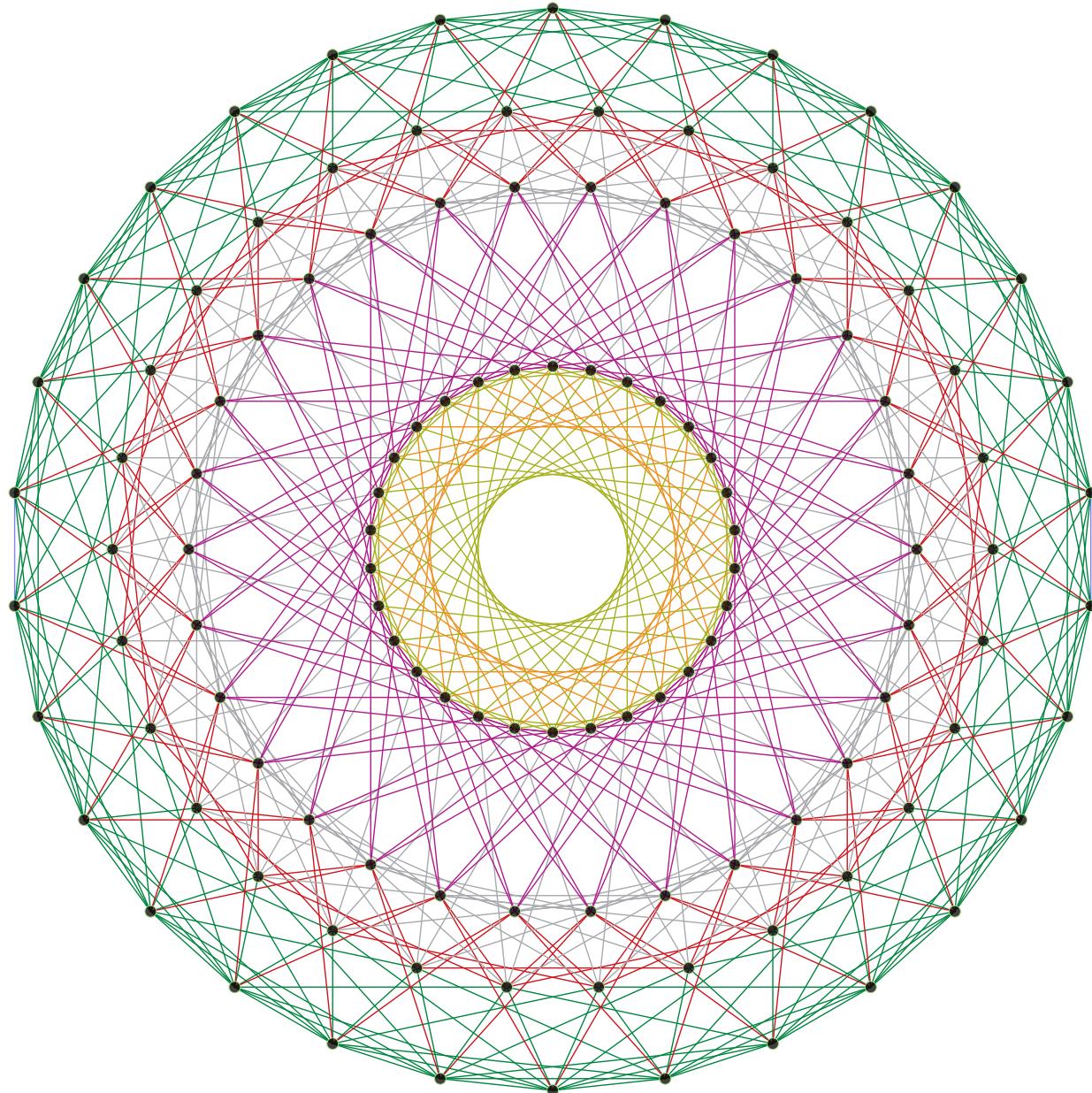


Figure 9: 4D \rightarrow 3D orthographic projection of the 600-Cell to a (perspective) virtual 3D object, with associated vertex shape, size, color/shade assigned based on extended Standard Model particle assignments

Note: If viewing *Mathematica*TM Computable Document Format (.CDF), this figure is interactive with zoom, rotate and vertex detail information on mouse-over.

Figure 10: 4D \rightarrow 2D Van Oss projection of the 600-Cell, the shadow of the projection in Figure 9

Note: If viewing *Mathematica*TM Computable Document Format (.CDF), this figure is interactive with zoom and vertex detail information on mouse-over.

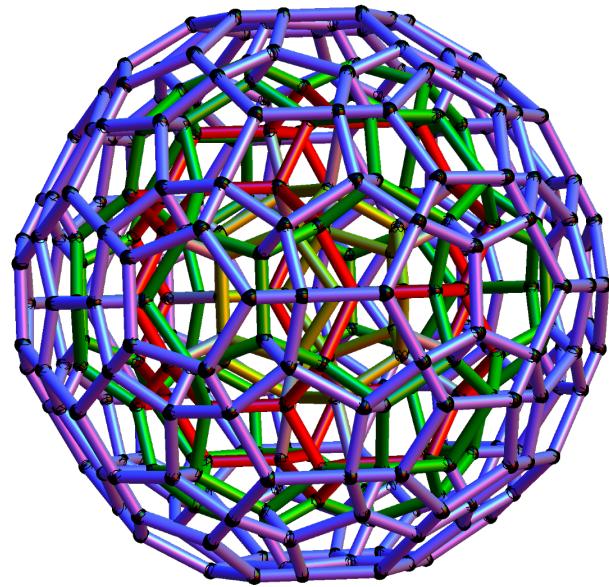


Figure 11: 4D \rightarrow 3D orthogonal ($H=\{1,0,0,0\}$, $V=\{0,1,0,0\}$, $Z=\{0,0,1,0\}\}$) orthographic projection of the 120-Cell to a virtual (perspective) 3D object

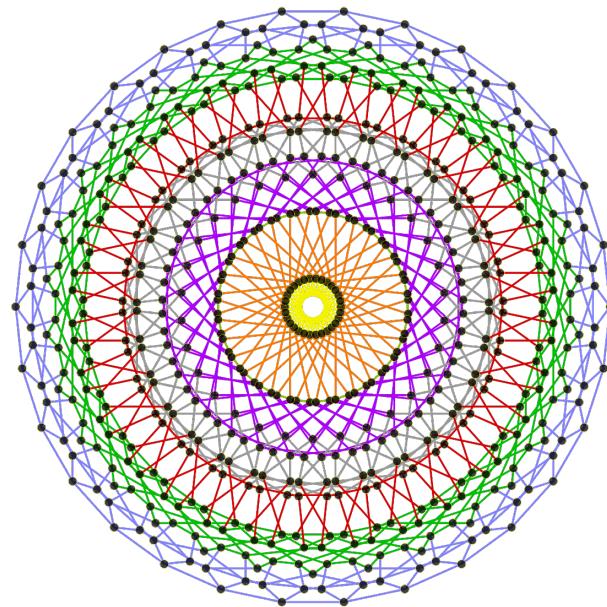
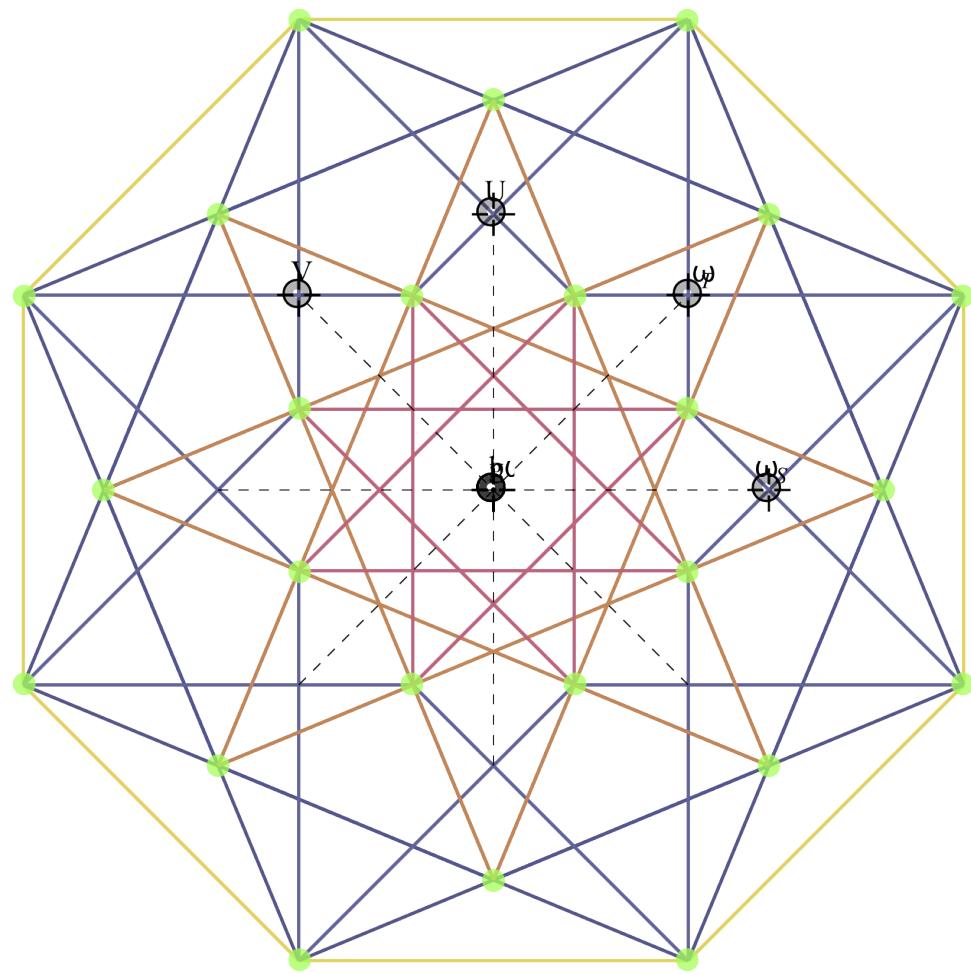
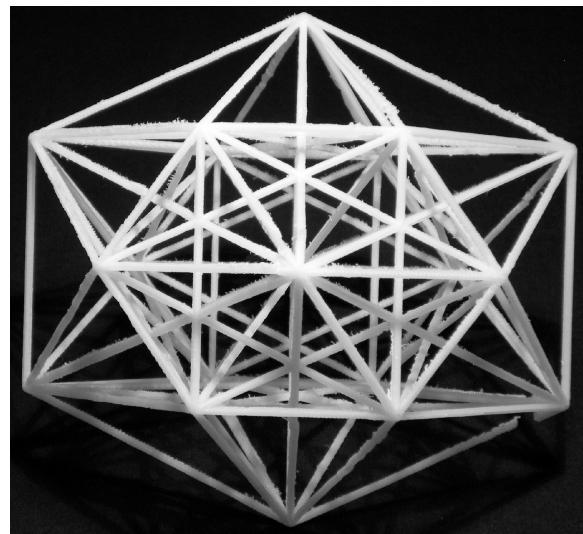
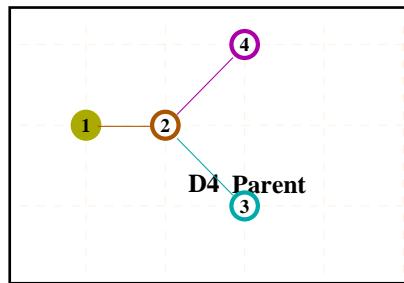
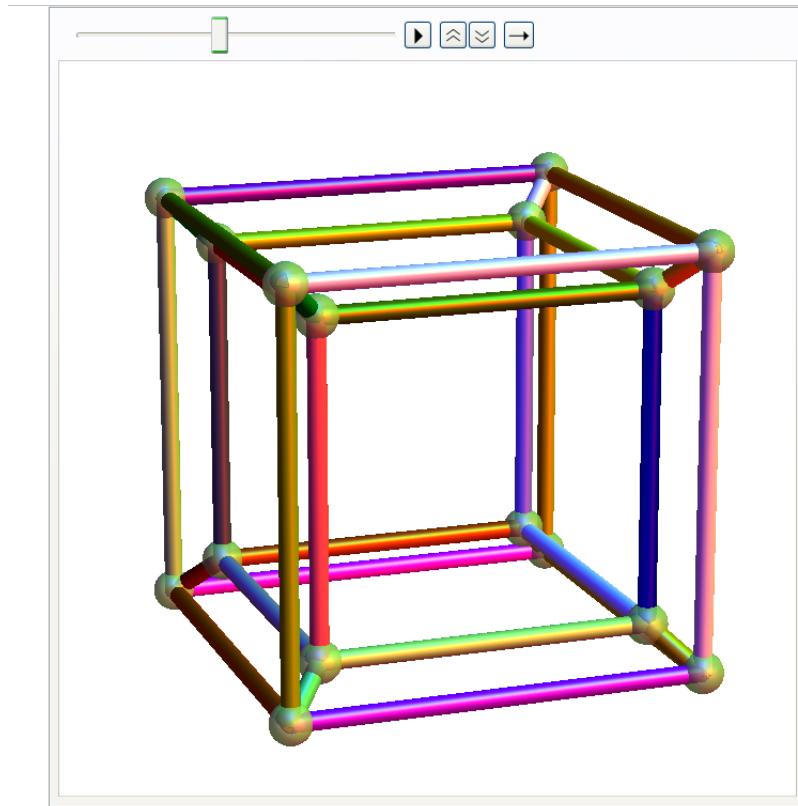


Figure 12: 4D \rightarrow 2D Petrie projection of the 120-Cell, which uses the basis vectors in (12)

Figure 13: 4D \rightarrow 2D orthographic projection of the 24-Cell, with basis vectors $\{\omega_S, i\omega_T, U, V\}$ shown

Note: If viewing MathematicaTM Computable Document Format (.CDF), this figure is interactive with vertex detail information on mouse-over.

Figure 14: 2D photo of a 4D \rightarrow 3D orthographic projection of the 24-Cell Laser-Stereolithographically grown as a plastic (perspective) 3D object

Figure 15: D_4 Dynkin diagramFigure 16: 4D→3D perspective projection of the Tesseract with 4D camera location very close to the outer perimeter of the object instead of at ∞ , as is done in orthographic projections. It is shown as a virtual (perspective) 3D object. The animation rotates in 4D and projects using the same basis vectors as Figure 13.

Note: If viewing MathematicaTM Computable Document Format (.CDF), this figure is interactive with 4D perspective rotation animations, with individual frame zoom and rotate controls.

The 600-Cell in Figure 9 was orthographically projected from 4D into 3D with basis vectors in (12), which has 720 shortest 4D edges of norm'd length $\sqrt{2} \left(-1 + \sqrt{5} \right)$. Notice the parallax due to the fact that it is displayed as a (perspective) 3D object. The orthographic shadow of this polytope is the Van Oss projection (Figure 10). Evidence of the folding is obtained by combining two 600-Cells at the golden ratio. This results in a structure that is somewhat isomorphic to E_8 [3]. It has the same Petrie projection as Figure 2, except with 3360 edges of length $\sqrt{2} \left(-1 + \sqrt{5} \right)$, which are no longer the shortest edges. This is half of the 6720 E_8 shortest edges of 8D norm'd length $\sqrt{2}$. The "other half" of the 6720 edges are the 3360 4D norm'd length $2\sqrt{2}$ edges. Yet, it is this 1:1 correspondence with the E_8 Petrie projection that allows the SRE particle assignments to be extended to the golden ratio combination of two 600-Cells.

$$\begin{aligned}
 H &= \left\{ \phi \sqrt{\frac{1}{2} - \frac{1}{30} \sqrt{75 + 30\sqrt{5}}} \sin\left[\frac{\pi}{15}\right], 0, \sqrt{\frac{1}{2} - \frac{1}{30} \sqrt{75 + 30\sqrt{5}}} \sin\left[\frac{2\pi}{15}\right], 0 \right\} = 2 \{0, -0.0801064775214, 0, 0.236818395103\} \\
 V &= \left\{ 0, \frac{1}{2} \sqrt{\frac{1}{2} - \frac{1}{30} \sqrt{75 + 30\sqrt{5}}}, 0, \phi \sqrt{\frac{1}{2} - \frac{1}{30} \sqrt{75 + 30\sqrt{5}}} \sin\left[\frac{\pi}{30}\right] \right\} = 2 \{0.159335291712, 0, 0.192645438086, 0\} \\
 Z &= \left\{ 0, -\phi \sqrt{\frac{1}{2} - \frac{1}{30} \sqrt{75 + 30\sqrt{5}}} \sin\left[\frac{\pi}{30}\right], 0, \frac{1}{2} \sqrt{\frac{1}{2} - \frac{1}{30} \sqrt{75 + 30\sqrt{5}}} \right\} = 2 \{0, 0.236818395103, 0, 0.0801064775214\}
 \end{aligned} \tag{12}$$

Simple Root Particles

These 3 bits $\{a, p, g0\}$ are used to uniquely identify and generate the $8 = 2^3$ Dynkin diagram nodes as shown in Figure 17 with corresponding 3bit value construction and ordering identified along with the simple root particle assignments.

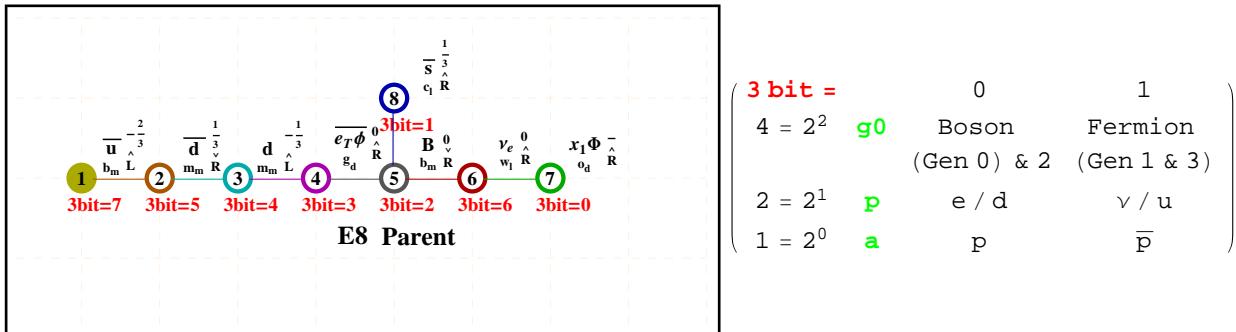


Figure 17: E_8 Dynkin diagram with particle labels and 3bit assignments and construction patterns

Note: If viewing MathematicaTM Computable Document Format (.CDF), this figure is interactive with node detail information on mouse-over.

Please note the seeming anomalous assignment of 3bit=1 to the last SR in the list of SRs (E_8 canonical node #8 in Figure 16, a \hat{R} anti-strange green quark). One might expect it to be a 5 given that it is a fermion, as indicated by the right most column naming for the $\{g0\}$ bit. While $\{g0\}$ does indeed split the generation 0 boson family of integer roots from the 1st and 3rd fermion generations, the naming of the 0 bit as "Boson" is somewhat of a misnomer since the 4 generations given by the combination of "generation bits" $\{g1, g0\}$ have $g0=0$ for generations 0 and 2 (in binary), such that they are associated through their bitwise assignments with bosons. It should also be noted that except for this E_8 canonical node #8, all fermions in the SRs are in the 1st generation and therefore have integer SRE vertex assignments. So all SRs are linked to bosons in some way. For a complete reference of SRs particle assignment detail, see Appendix A.

Triality Relationships

The Lisi model also demonstrates a consistency with the bosons and fermions that is related to the triality relationships within E_8 . This is shown in Figure 18 with blue triality lines linking the 3 generations of each fermion using (11). Applying the triality rotation matrix in (11) as a dot product against an SRE vector gives the 2nd generation fermion particle. Applying it again gives the 3rd generation. Applying it a 3rd time returns to the 1st generation fermion. The bosons are also involved in triality relationships as well using (10), rotating through red, green, and blue particle color assignments.

It is interesting to note that the quarks {r/g/b, p/ \bar{p} } are all located on 6 corresponding dual concentric circles around the center. The leptons are hexagonal "Star of David" patterns in the center, while the bosons are in single or dual hexagonal rings radiating from the center.

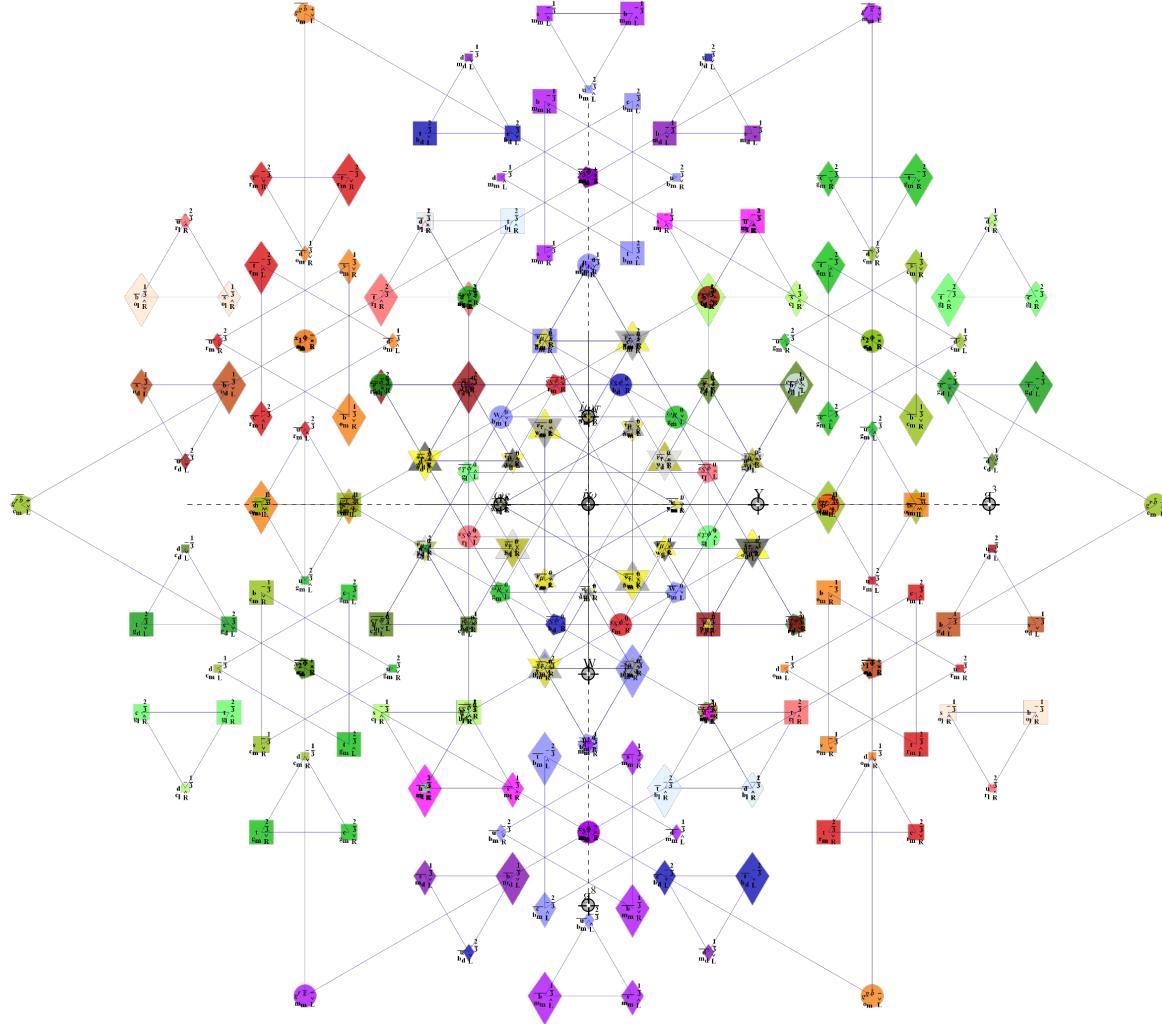


Figure 18: 8D \rightarrow 2D orthographic projection of the physics rotated SRE math coordinates of E_8 , with 86=22 bosonic+64 fermionic triality generated equilateral triangles. Vertex shape, size, color/shade are assigned based on extended Standard Model particle assignments.

$$\begin{aligned}
 H &= \left\{ 2 - \frac{4}{\sqrt{3}}, 0, 1 - \frac{1}{\sqrt{3}}, 1 - \frac{1}{\sqrt{3}}, 0, -1, 1, 0 \right\} \\
 V &= \left\{ 0, \frac{4}{\sqrt{3}} - 2, \frac{1}{\sqrt{3}} - 1, 1 - \frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}} \right\}
 \end{aligned} \tag{13}$$

The 8D \rightarrow 2D orthographic projections in Figures 18 and 19 are produced using SRE basis vectors (13). The axes shown in Figure 18 are rotated to physics coordinates using (9), which puts the basis vectors on the projected (H)orizontal and (V)ertical axes in this and the E_8 Petrie projections of Figures 1, 2, and 22 (Appendix D). It seems to clarify dimensional identities as well. For example, when the {1, 2, 3} dimensions labeled $\{\omega_S, i\omega_T, W\}$ are moved, all vertices change positions except the p-Type=0 bosons $\{g \text{ gluons}, x_n\Phi\}$. Moving the dimension {4} labeled {Y} preserves these as well as the \hat{L} and \hat{R} quark positions. Moving the {5,6} dimensions labeled $\{i\omega, x\}$ preserves these, except now the row 4 p-Type=0 bosons $\{x_n\Phi\}$ emerge from the 6 triple overlap points at center of the quark's concentric rings (the intersection of the gluons triality lines). And finally {7, 8} labeled $\{g_3, g_8\}$ in physics can be identified with quark color, as $\{g_3\}$ preserves the blue quark positions, while $\{g_8\}$ moves the dual concentric rings of quarks while preserving their relative positions within the rings. It is interesting to note that the dimensions {6, 7, 8} are appropriately labeled {r, g, b} in SRE coordinates, since in this projection the SRE math coordinates are located at the aforementioned 6 triple overlap points at center of the quark's {r& \bar{r} , g& \bar{g} , b& \bar{b} } concentric rings (the intersection of the gluons triality lines).

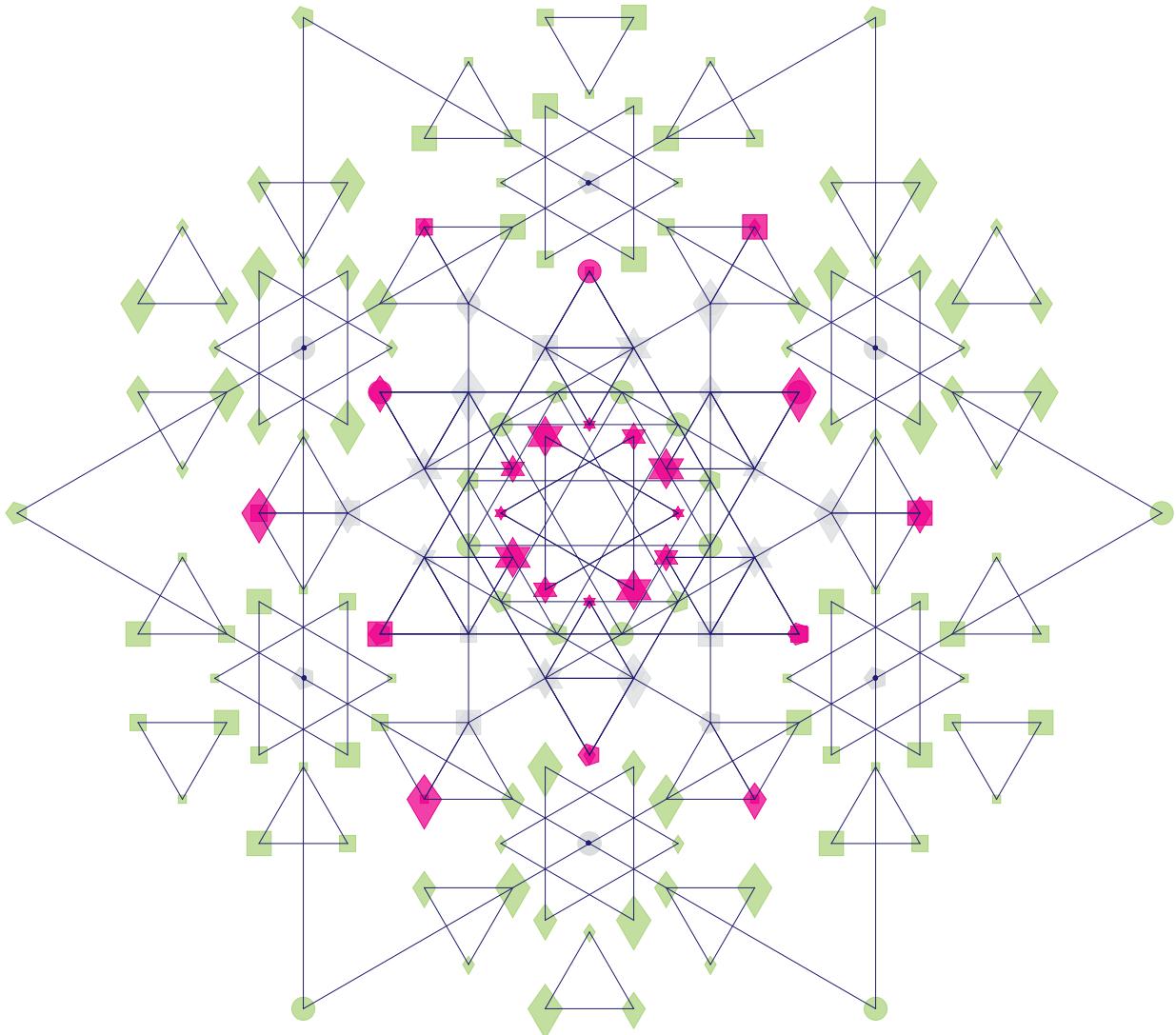


Figure 19: Projection of Figure 18 with particles color coded for overlaps

Note: If viewing *Mathematica*TM Computable Document Format (.CDF), this figure is interactive with vertex detail information on mouse-over.

Looking carefully at the color coding of the trialities in Figure 18, the smaller 1st generation quarks are shaded differently than the 2nd and 3rd generation quarks. This is due to a change in the way the {**p**} bit is used to assign patterns to SRE vertices on 1st generation quarks. This change is needed to align the 3bit numbers in 1:1 correspondence with the Dynkin nodes. It should be noted that this also aligns with the quark mass relationships in the SM, where the 1st generation up/down quark masses are flipped.

$$\begin{aligned}
 \text{InView vertices} &= \{\text{Color}[\text{overlap}, \text{count}], \dots\} \text{ Total} \\
 &\{\text{Green}\{1, 120\}, \text{Red}\{2, 24\}, \text{Gray}\{3, 24\}\} 168 \\
 \text{All vertices} &= \{\text{Color}[\text{overlap}, \text{count}], \dots\} \text{ Total} \\
 &\{\text{Green}\{1, 120\}, \text{Red}\{2, 48\}, \text{Gray}\{3, 72\}\} 240
 \end{aligned} \tag{14}$$

An interesting pattern of overlapping of particles (14) in this projection is shown in Figure 19.

Conclusion

This mixture of fermion assignments with bosons is a critical issue in Lisi's model. Therefore, this new binary model may shed some light on the solution, along with the mass determination in extended SM physics. A natural choice would be to assign 2nd generation fermions to the integer SRE vertices as Lisi did originally, but this causes the 3bit calculations to lack a 1:1 relationship with the Dynkin diagram's SRs.

Interestingly, by taking account of the particle mass assignments, all known fermions {e/ν, u/d, c/s, t/b}, as well as known (plus the Lisi predicted) bosons {W/B, gluons(g), ω, eΦ, xΦ} can be generated with the sum of the simple root masses being less than the resulting composite particle masses, with the exception of the four 2nd and 3rd generation leptons {eμ,τ / νμ,τ}.

It seems logical to identify the SRs as elemental particles used to construct the known and predicted 240 fundamental particles.

Given the overlap of 120 and 24 vertex patterns in (14), it seems the elemental particles may be related to the number of self dual 24-Cell structures occurring from left and right 4D vertex dimensions of E_8 . If this is the case, the prediction of the particle masses and the convergence of General Relativity (GR) and an extended SM may be a reasonable result. That is, a grand unification - a Theory of Everything (ToE).

References

- [1] A. G. Lisi, *An exceptionally simple theory of everything* (2007), URL <http://www.citebase.org/abstract?id=oai:arXiv.org:0711.0770>.
- [2] Jacques Distler; Skip Garibaldi (2009). *There is no 'Theory of Everything' inside E8*. URL <http://www.citebase.org/abstract?id=oai:arXiv.org:0905.2658>.
- [3] David A. Richter (2007). *Triacantagonal coordinates for the E(8) root system*. URL <http://www.citebase.org/abstract?id=oai:arXiv.org:0704.3091>.

Appendix

Appendix A: Simple Roots List

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
58	$\overline{u} \begin{smallmatrix} 2 \\ -3 \\ \wedge \\ b_m \\ L \end{smallmatrix}$	011111001 ₂	{1, 0, 0, 0, 0, 0, 1, 1}	{0, -1, 0, 0, 0, 0, 0, 1}	{1, 0, 0, 0, 0, 0, 0, 0}
200	$\overline{d} \begin{smallmatrix} 1 \\ 3 \\ \vee \\ m_m \\ R \end{smallmatrix}$	010111101 ₂	{0, 1, 1, 1, 1, 0, 1, 0}	{0, 1, 0, 0, 0, 0, 0, 1}	{0, 1, 0, 0, 0, 0, 0, 0}
212	$d \begin{smallmatrix} 1 \\ -3 \\ \wedge \\ m_m \\ L \end{smallmatrix}$	000111001 ₂	{0, 1, 0, 1, 0, 1, 1, 1}	{1, 0, 0, 0, 0, 0, 0, -1}	{0, 0, 1, 0, 0, 0, 0, 0}
50	$\overline{e_T \phi} \begin{smallmatrix} 0 \\ g_a \\ R \end{smallmatrix}$	011100100 ₂	{1, 0, 0, 1, 0, 1, 0, 0}	{-1, 0, 1, 0, 0, 0, 0, 0}	{0, 0, 0, 1, 0, 0, 0, 0}
73	$B \begin{smallmatrix} 0 \\ b_m \\ R \end{smallmatrix}$	001111100 ₂	{0, 1, 0, 0, 0, 0, 1, 1}	{0, 0, -1, 1, 0, 0, 0, 0}	{0, 0, 0, 0, 1, 0, 0, 0}
81	$\nu_e \begin{smallmatrix} 0 \\ w_1 \\ R \end{smallmatrix}$	001000101 ₂	{0, 0, 1, 0, 0, 1, 1, 0}	{0, 0, 0, -1, 1, 0, 0, 0}	{0, 0, 0, 0, 0, 1, 0, 0}
87	$x_1 \Phi \begin{smallmatrix} - \\ o_a \\ R \end{smallmatrix}$	000010100 ₂	{0, 0, 0, 1, 0, 1, 1, 0}	{0, 0, 0, 0, -1, 1, 0, 0}	{0, 0, 0, 0, 0, 0, 1, 0}
21	$\overline{s} \begin{smallmatrix} 1 \\ 3 \\ c_1 \\ R \end{smallmatrix}$	010100110 ₂	{0, 1, 0, 0, 0, 0, 1, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{0, 0, 0, 0, 0, 0, 0, 1}

Appendix B: Complete Particle List

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
1	$\nu_{\tau}^0_{w_1 \hat{R}}$	001000111 ₂	{0, 0, 0, 0, 0, 0, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-3, -3, -5, -4, -3, -2, -1, -1}
Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
2	$\overline{Ex1}^+_{\overset{\vee}{L}}$	010000000 ₂	{1, 0, 0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0, 0, -1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0, 0\right\}$
3	$\overline{Ex1}^+_{\overset{\vee}{R}}$	010000100 ₂	{0, 1, 0, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, 0, -1, 0}	$\left\{-\frac{7}{2}, -\frac{5}{2}, -5, -4, -3, -2, -1, -2\right\}$
4	$\overline{Ex1}^+_{\overset{\vee}{L}}$	010001000 ₂	{0, 0, 1, 0, 0, 0, 0, 0}	{0, 0, 0, 0, 0, -1, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -1, -1, -1, -1, -1, 0\right\}$
5	$\overline{Ex1}^+_{\overset{\vee}{R}}$	010001100 ₂	{0, 0, 0, 1, 0, 0, 0, 0}	{0, 0, 0, 0, -1, 0, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -1, -1, -1, -1, 0, 0\right\}$
6	$\overline{Ex2}^0_{\overset{\vee}{L}}$	011000000 ₂	{0, 0, 0, 0, 1, 0, 0, 0}	{0, 0, 0, -1, 0, 0, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -1, -1, -1, -1, 0, 0, 0\right\}$
7	$\overline{Ex2}^0_{\overset{\vee}{R}}$	011000100 ₂	{0, 0, 0, 0, 0, 1, 0, 0}	{0, 0, -1, 0, 0, 0, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -1, -1, 0, 0, 0, 0\right\}$
8	$\overline{Ex2}^0_{\hat{L}}$	011001000 ₂	{0, 0, 0, 0, 0, 0, 1, 0}	{0, -1, 0, 0, 0, 0, 0, 0}	$\left\{\frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0, 0\right\}$
9	$\overline{Ex2}^0_{\hat{R}}$	011001100 ₂	{0, 0, 0, 0, 0, 0, 0, 1}	{-1, 0, 0, 0, 0, 0, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -1, 0, 0, 0, 0\right\}$
Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
10	$\nu_{\tau}^0_{w_d \hat{L}}$	001000011 ₂	{1, 1, 0, 0, 0, 0, 0, 0}	$\left\{\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-3, -2, -4, -4, -3, -2, -1, -1}
11	$\nu_{\tau}^0_{w_m \hat{R}}$	001001111 ₂	{1, 0, 1, 0, 0, 0, 0, 0}	$\left\{\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-2, -2, -3, -3, -3, -2, -1, -1}
12	$e_{\tau}^-_{y_m \hat{R}}$	000001111 ₂	{1, 0, 0, 1, 0, 0, 0, 0}	$\left\{\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-2, -2, -3, -3, -2, -2, -1, -1}
13	$\overline{e}_{\mu}^+_{y_d \hat{L}}$	010000010 ₂	{1, 0, 0, 0, 1, 0, 0, 0}	$\left\{\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-2, -2, -3, -3, -2, -1, -1, -1}

14	$\overline{\mathbf{s}}_{\text{d}}^{\frac{1}{3}}$	010010010 ₂	{1, 0, 0, 0, 0, 1, 0, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{-2, -2, -3, -3, -2, -1, 0, -1}
15	$\overline{\mathbf{s}}_{\text{d}}^{\frac{1}{3}}$	010100010 ₂	{1, 0, 0, 0, 0, 0, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{1, 0, 1, 0, 0, 0, 0, 1}
16	$\overline{\mathbf{s}}_{\text{m}}^{\frac{1}{3}}$	010110010 ₂	{1, 0, 0, 0, 0, 0, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{-2, -2, -4, -4, -3, -2, -1, -1}
17	$\nu_{\tau}^0_{w_m \hat{L}}$	001001011 ₂	{0, 1, 1, 0, 0, 0, 0, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-3, -2, -4, -3, -3, -2, -1, -1}
18	$e_{\tau}^{-}_{Y_m \hat{L}}$	000001011 ₂	{0, 1, 0, 1, 0, 0, 0, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-3, -2, -4, -3, -2, -2, -1, -1}
19	$\overline{e}_{\mu}^{+}_{Y_1 \hat{R}}$	010000110 ₂	{0, 1, 0, 0, 1, 0, 0, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-3, -2, -4, -3, -2, -1, -1, -1}
20	$\overline{\mathbf{s}}_{o_1 \hat{R}}^{\frac{1}{3}}$	010010110 ₂	{0, 1, 0, 0, 0, 1, 0, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-3, -2, -4, -3, -2, -1, 0, -1}
21	$\overline{\mathbf{s}}_{c_1 \hat{R}}^{\frac{1}{3}}$	010100110 ₂	{0, 1, 0, 0, 0, 0, 1, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{0, 0, 0, 0, 0, 0, 0, 1}
22	$\overline{\mathbf{s}}_{m_1 \hat{R}}^{\frac{1}{3}}$	010110110 ₂	{0, 1, 0, 0, 0, 0, 0, 1}	$\left\{ -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{-3, -2, -5, -4, -3, -2, -1, -1}
23	$e_{\tau}^{-}_{Y_1 \hat{R}}$	000000111 ₂	{0, 0, 1, 1, 0, 0, 0, 0}	$\left\{ -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-2, -2, -3, -2, -2, -2, -1, -1}
24	$\overline{e}_{\mu}^{+}_{Y_m \hat{L}}$	010001010 ₂	{0, 0, 1, 0, 1, 0, 0, 0}	$\left\{ -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-2, -2, -3, -2, -2, -1, -1, -1}
25	$\overline{\mathbf{s}}_{o_m \hat{L}}^{\frac{1}{3}}$	010011010 ₂	{0, 0, 1, 0, 0, 1, 0, 0}	$\left\{ -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{-2, -2, -3, -2, -2, -1, 0, -1}
26	$\overline{\mathbf{s}}_{c_m \hat{L}}^{\frac{1}{3}}$	010101010 ₂	{0, 0, 1, 0, 0, 0, 1, 0}	$\left\{ -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{1, 0, 1, 1, 0, 0, 0, 1}
27	$\overline{\mathbf{s}}_{m \hat{L}}^{\frac{1}{3}}$	010111010 ₂	{0, 0, 1, 0, 0, 0, 0, 1}	$\left\{ -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{-2, -2, -4, -3, -3, -2, -1, -1}

28	$\overline{e}_\mu \overset{+}{\underset{\text{Y}_m}{R}}$	010001110 ₂	{0, 0, 0, 1, 1, 0, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-2, -2, -3, -2, -1, -1, -1, -1}
29	$\overline{s} \overset{1}{\underset{\text{o}_m}{R}}$	010011110 ₂	{0, 0, 0, 1, 0, 1, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-2, -2, -3, -2, -1, -1, 0, -1}
30	$\overline{s} \overset{1}{\underset{\text{c}_m}{R}}$	010101110 ₂	{0, 0, 0, 1, 0, 0, 1, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{1, 0, 1, 1, 1, 0, 0, 1}
31	$\overline{s} \overset{1}{\underset{\text{m}_m}{R}}$	010111110 ₂	{0, 0, 0, 1, 0, 0, 0, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-2, -2, -4, -3, -2, -2, -1, -1}
32	$\overline{b} \overset{1}{\underset{\text{o}_d}{L}}$	010010011 ₂	{0, 0, 0, 0, 1, 1, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-2, -2, -3, -2, -1, 0, 0, -1}
33	$\overline{b} \overset{1}{\underset{\text{c}_d}{L}}$	010100011 ₂	{0, 0, 0, 0, 1, 0, 1, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{1, 0, 1, 1, 1, 1, 0, 1}
34	$\overline{b} \overset{1}{\underset{\text{m}_d}{L}}$	010110011 ₂	{0, 0, 0, 0, 1, 0, 0, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	{-2, -2, -4, -3, -2, -1, -1, -1}
35	$t \overset{2}{\underset{\text{b}_1}{R}}$	001110111 ₂	{0, 0, 0, 0, 0, 1, 1, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{1, 0, 1, 1, 1, 1, 1, 1}
36	$t \overset{2}{\underset{\text{g}_1}{R}}$	001100111 ₂	{0, 0, 0, 0, 0, 1, 0, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-2, -2, -4, -3, -2, -1, 0, -1}
37	$t \overset{2}{\underset{\text{r}_1}{R}}$	001010111 ₂	{0, 0, 0, 0, 0, 0, 1, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	{1, 0, 0, 0, 0, 0, 0, 1}

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
38	$\overline{w} \overset{0}{\underset{\text{b}_m}{L}}$	011110000 ₂	{1, 1, 1, 0, 0, 0, 0, 0}	{-1, -1, 0, 0, 0, 0, 0, 0}	{0, -1, -1, 0, 0, 0, 0, 0}
39	$\overline{e}_S \phi \overset{0}{\underset{\text{r}_m}{R}}$	011011100 ₂	{1, 1, 0, 1, 0, 0, 0, 0}	{-1, 0, -1, 0, 0, 0, 0, 0}	{-1, -1, -2, -1, 0, 0, 0, 0}
40	$\overline{e}_S \phi \overset{0}{\underset{\text{b}_d}{R}}$	011110100 ₂	{1, 1, 0, 0, 1, 0, 0, 0}	{-1, 0, 0, -1, 0, 0, 0, 0}	{-1, -1, -2, -1, -1, 0, 0, 0}
41	$\overline{v}_e \overset{0}{\underset{\text{w}_m}{L}}$	011001001 ₂	{1, 1, 0, 0, 0, 1, 0, 0}	{-1, 0, 0, 0, -1, 0, 0, 0}	{-1, -1, -2, -1, -1, -1, 0, 0}

42	$\overset{2}{\underset{\text{r}_m \text{ R}}{u}}$	001011101_2	$\{1, 1, 0, 0,$ $0, 0, 1, 0\}$	$\{-1, 0, 0, 0,$ $0, -1, 0, 0\}$	$\{-1, -1, -2, -1,$ $-1, -1, -1, 0\}$
43	$\overset{2}{\underset{\text{g}_m \text{ R}}{u}}$	001101101_2	$\{1, 1, 0, 0,$ $0, 0, 0, 1\}$	$\{-1, 0, 0, 0,$ $0, 0, -1, 0\}$	$\{-4, -3, -6, -4,$ $-3, -2, -1, -2\}$
44	$\overset{2}{\underset{\text{b}_m \text{ R}}{u}}$	001111101_2	$\{1, 0, 1, 1,$ $0, 0, 0, 0\}$	$\{-1, 0, 0, 0,$ $0, 0, 0, -1\}$	$\{-1, -1, -1,$ $0, 0, 0, 0, 0\}$
45	$\overset{1}{\underset{\text{m}_m \text{ L}}{\bar{d}}}$	010111001_2	$\{1, 0, 1, 0,$ $1, 0, 0, 0\}$	$\{-1, 0, 0,$ $0, 0, 0, 0, 1\}$	$\{0, 0, -1,$ $0, 0, 0, 0, 0\}$
46	$\overset{1}{\underset{\text{c}_m \text{ L}}{\bar{d}}}$	010101001_2	$\{1, 0, 1, 0,$ $0, 1, 0, 0\}$	$\{-1, 0, 0,$ $0, 0, 0, 1, 0\}$	$\{3, 2, 4, 4,$ $3, 2, 1, 2\}$
47	$\overset{1}{\underset{\text{o}_m \text{ L}}{\bar{d}}}$	010011001_2	$\{1, 0, 1, 0,$ $0, 0, 1, 0\}$	$\{-1, 0, 0,$ $0, 0, 1, 0, 0\}$	$\{0, 0, 0, 1,$ $1, 1, 1, 0\}$
48	$\overset{-}{\underset{\text{y}_m \text{ R}}{e}}$	000001101_2	$\{1, 0, 1, 0,$ $0, 0, 0, 1\}$	$\{-1, 0, 0,$ $0, 1, 0, 0, 0\}$	$\{0, 0, 0, 1,$ $1, 1, 0, 0\}$
49	$\overset{0}{\underset{\text{b}_1 \text{ L}}{e_T \phi}}$	001111000_2	$\{1, 0, 0, 1,$ $1, 0, 0, 0\}$	$\{-1, 0, 0,$ $1, 0, 0, 0, 0\}$	$\{0, 0, 0, 1,$ $1, 0, 0, 0\}$
50	$\overset{0}{\underset{\text{g}_d \text{ R}}{\bar{e}_T \phi}}$	011100100_2	$\{1, 0, 0, 1,$ $0, 1, 0, 0\}$	$\{-1, 0, 1,$ $0, 0, 0, 0, 0\}$	$\{0, 0, 0, 1,$ $0, 0, 0, 0\}$
51	$\overset{0}{\underset{\text{g}_m \text{ L}}{\omega_R}}$	001100000_2	$\{1, 0, 0, 1,$ $0, 0, 1, 0\}$	$\{-1, 1, 0,$ $0, 0, 0, 0, 0\}$	$\{-1, 0, -1,$ $0, 0, 0, 0, 0\}$
52	$\overset{0}{\underset{\text{g}_1 \text{ L}}{\bar{e}_T \phi}}$	011101000_2	$\{1, 0, 0, 1,$ $0, 0, 0, 1\}$	$\{0, -1, -1,$ $0, 0, 0, 0, 0\}$	$\{0, -1, -1,$ $-1, 0, 0, 0, 0\}$
53	$\overset{0}{\underset{\text{r}_d \text{ R}}{\bar{e}_S \phi}}$	011010100_2	$\{1, 0, 0, 0,$ $1, 1, 0, 0\}$	$\{0, -1, 0,$ $-1, 0, 0, 0, 0\}$	$\{0, -1, -1, -1,$ $-1, 0, 0, 0\}$
54	$\overset{+}{\underset{\text{y}_m \text{ L}}{\bar{e}}}$	010001001_2	$\{1, 0, 0, 0,$ $1, 0, 1, 0\}$	$\{0, -1, 0, 0,$ $-1, 0, 0, 0\}$	$\{0, -1, -1, -1,$ $-1, -1, 0, 0\}$
55	$\overset{-1}{\underset{\text{o}_m \text{ R}}{d}}$	000011101_2	$\{1, 0, 0, 0,$ $1, 0, 0, 1\}$	$\{0, -1, 0, 0,$ $0, -1, 0, 0\}$	$\{0, -1, -1, -1,$ $-1, -1, -1, 0\}$
56	$\overset{-1}{\underset{\text{c}_m \text{ R}}{d}}$	000101101_2	$\{1, 0, 0, 0,$ $0, 1, 1, 0\}$	$\{0, -1, 0, 0,$ $0, 0, -1, 0\}$	$\{-3, -3, -5, -4,$ $-3, -2, -1, -2\}$
57	$\overset{-1}{\underset{\text{m}_m \text{ R}}{d}}$	000111101_2	$\{1, 0, 0, 0,$ $0, 1, 0, 1\}$	$\{0, -1, 0, 0,$ $0, 0, 0, -1\}$	$\{0, -1, 0,$ $0, 0, 0, 0, 0\}$
58	$\overset{2}{\underset{\text{b}_m \text{ L}}{\bar{u}}}$	011111001_2	$\{1, 0, 0, 0,$ $0, 0, 1, 1\}$	$\{0, -1, 0,$ $0, 0, 0, 0, 1\}$	$\{1, 0, 0, 0,$ $0, 0, 0, 0, 0\}$

59	$\overline{u}_{g_m}^{\frac{2}{\lambda}}$	011101001 ₂	{0, 1, 1, 1, 0, 0, 0, 0}	{0, -1, 0, 0, 0, 1, 0}	{4, 2, 5, 4, 3, 2, 1, 2}
60	$\overline{u}_{r_m}^{\frac{2}{\lambda}}$	011011001 ₂	{0, 1, 1, 0, 1, 0, 0, 0}	{0, -1, 0, 0, 0, 1, 0, 0}	{1, 0, 1, 1, 1, 1, 1, 0}
61	$\nu_e_{w_m}^0$	0010001101 ₂	{0, 1, 1, 0, 0, 1, 0, 0}	{0, -1, 0, 0, 1, 0, 0, 0}	{1, 0, 1, 1, 1, 1, 0, 0}
62	$\overline{e}_s \phi_{r_1}^0$	011011000 ₂	{0, 1, 1, 0, 0, 0, 1, 0}	{0, -1, 0, 1, 0, 0, 0, 0}	{1, 0, 1, 1, 1, 0, 0, 0}
63	$\overline{e}_T \phi_{g_m}^0$	011101100 ₂	{0, 1, 1, 0, 0, 0, 0, 1}	{0, -1, 1, 0, 0, 0, 0, 0}	{1, 0, 1, 1, 0, 0, 0, 0}
64	\overline{w}_L^0	011010000 ₂	{0, 1, 0, 1, 1, 0, 0, 0}	{0, 0, -1, -1, 0, 0, 0, 0}	{-1, -1, -2, -2, -1, 0, 0, 0}
65	$\overline{e}_{y_1}^+$	010000101 ₂	{0, 1, 0, 1, 0, 1, 0, 0}	{0, 0, -1, 0, -1, 0, 0, 0}	{-1, -1, -2, -2, -1, -1, 0, 0}
66	$d_{o_d}^{\frac{1}{\lambda}}$	000010001 ₂	{0, 1, 0, 1, 0, 0, 1, 0}	{0, 0, -1, 0, 0, -1, 0, 0}	{-1, -1, -2, -2, -1, -1, -1, 0}
67	$d_{c_d}^{\frac{1}{\lambda}}$	000100001 ₂	{0, 1, 0, 1, 0, 0, 0, 1}	{0, 0, -1, 0, 0, 0, -1, 0}	{-4, -3, -6, -5, -3, -2, -1, -2}
68	$d_{m_d}^{\frac{1}{\lambda}}$	000110001 ₂	{0, 1, 0, 0, 1, 1, 0, 0}	{0, 0, -1, 0, 0, 0, 0, -1}	{-1, -1, -1, -1, 0, 0, 0, 0}
69	$\overline{u}_{b_1}^{\frac{2}{\lambda}}$	011110101 ₂	{0, 1, 0, 0, 1, 0, 1, 0}	{0, 0, -1, 0, 0, 0, 0, 1}	{0, 0, -1, -1, 0, 0, 0, 0}
70	$\overline{u}_{g_1}^{\frac{2}{\lambda}}$	011100101 ₂	{0, 1, 0, 0, 1, 0, 0, 1}	{0, 0, -1, 0, 0, 0, 1, 0}	{3, 2, 4, 3, 3, 2, 1, 2}
71	$\overline{u}_{r_1}^{\frac{2}{\lambda}}$	011010101 ₂	{0, 1, 0, 0, 0, 1, 1, 0}	{0, 0, -1, 0, 0, 1, 0, 0}	{0, 0, 0, 0, 1, 1, 1, 0}
72	$\nu_e_{w_d}^0$	001000001 ₂	{0, 1, 0, 0, 0, 1, 0, 1}	{0, 0, -1, 0, 1, 0, 0, 0}	{0, 0, 0, 0, 1, 1, 0, 0}
73	$B_{b_m}^0$	001111100 ₂	{0, 1, 0, 0, 0, 0, 1, 1}	{0, 0, -1, 1, 0, 0, 0, 0}	{0, 0, 0, 0, 1, 0, 0, 0}
74	$\overline{e}_{y_d}^+$	010000001 ₂	{0, 0, 1, 1, 1, 0, 0, 0}	{0, 0, 0, -1, -1, 0, 0, 0}	{-1, -1, -2, -2, -2, -1, 0, 0}
75	$d_{o_1}^{\frac{1}{\lambda}}$	000010101 ₂	{0, 0, 1, 1, 0, 1, 0, 0}	{0, 0, 0, -1, 0, -1, 0, 0}	{-1, -1, -2, -2, -2, -1, -1, 0}
76	$d_{c_1}^{\frac{1}{\lambda}}$	000100101 ₂	{0, 0, 1, 1, 0, 0, 1, 0}	{0, 0, 0, -1, 0, 0, -1, 0}	{-4, -3, -6, -5, -4, -2, -1, -2}

77	$d_{m_1 \hat{R}}^{-\frac{1}{3}}$	000110101 ₂	{0, 0, 1, 1, 0, 0, 0, 1}	{0, 0, 0, -1, 0, 0, 0, -1}	{-1, -1, -1, -1, -1, 0, 0, 0}
78	$\overline{u}_{b_d \hat{L}}^{-\frac{2}{3}}$	011110001 ₂	{0, 0, 1, 0, 1, 1, 0, 0}	{0, 0, 0, -1, 0, 0, 0, 1}	{0, 0, -1, -1, -1, 0, 0, 0}
79	$\overline{u}_{g_d \hat{L}}^{-\frac{2}{3}}$	011100001 ₂	{0, 0, 1, 0, 1, 0, 1, 0}	{0, 0, 0, -1, 0, 0, 1, 0}	{3, 2, 4, 3, 2, 2, 1, 2}
80	$\overline{u}_{r_d \hat{L}}^{-\frac{2}{3}}$	011010001 ₂	{0, 0, 1, 0, 1, 0, 0, 1}	{0, 0, 0, -1, 0, 1, 0, 0}	{0, 0, 0, 0, 0, 1, 1, 0}
81	$v_e^0_{w_1 \hat{R}}$	001000101 ₂	{0, 0, 1, 0, 0, 1, 1, 0}	{0, 0, 0, -1, 1, 0, 0, 0}	{0, 0, 0, 0, 0, 1, 0, 0}
82	$\overline{x_1 \Phi}_{o_m \hat{R}}^+$	010011100 ₂	{0, 0, 1, 0, 0, 1, 0, 1}	{0, 0, 0, 0, -1, -1, 0, 0}	{-1, -1, -2, -2, -2, -2, -1, 0}
83	$\overline{x_2 \Phi}_{c_m \hat{R}}^+$	010101100 ₂	{0, 0, 1, 0, 0, 0, 1, 1}	{0, 0, 0, 0, -1, 0, -1, 0}	{-4, -3, -6, -5, -4, -3, -1, -2}
84	$\overline{x_3 \Phi}_{m_m \hat{R}}^+$	010111100 ₂	{0, 0, 0, 1, 1, 1, 0, 0}	{0, 0, 0, 0, -1, 0, 0, -1}	{-1, -1, -1, -1, -1, -1, 0, 0}
85	$x_3 \Phi_{m_d \hat{R}}^-$	000110100 ₂	{0, 0, 0, 1, 1, 0, 1, 0}	{0, 0, 0, 0, -1, 0, 0, 1}	{0, 0, -1, -1, -1, -1, 0, 0}
86	$x_2 \Phi_{c_d \hat{R}}^-$	000100100 ₂	{0, 0, 0, 1, 1, 0, 0, 1}	{0, 0, 0, 0, -1, 0, 1, 0}	{3, 2, 4, 3, 2, 1, 1, 2}
87	$x_1 \Phi_{o_d \hat{R}}^-$	000010100 ₂	{0, 0, 0, 1, 0, 1, 1, 0}	{0, 0, 0, 0, -1, 1, 0, 0}	{0, 0, 0, 0, 0, 0, 1, 0}
88	$x_3 \Phi_{m_1 \hat{L}}^-$	000111000 ₂	{0, 0, 0, 1, 0, 1, 0, 1}	{0, 0, 0, 0, 0, -1, -1, 0}	{-4, -3, -6, -5, -4, -3, -2, -2}
89	$x_2 \Phi_{c_1 \hat{L}}^-$	000101000 ₂	{0, 0, 0, 1, 0, 0, 1, 1}	{0, 0, 0, 0, 0, -1, 0, -1}	{-1, -1, -1, -1, -1, -1, -1, 0}
90	$g^g \bar{b}_{o_m \hat{L}}^-$	000010000 ₂	{0, 0, 0, 0, 1, 1, 1, 0}	{0, 0, 0, 0, 0, -1, 0, 1}	{0, 0, -1, -1, -1, -1, -1, 0}
91	$g^x \bar{b}_{c_m \hat{L}}^-$	000100000 ₂	{0, 0, 0, 0, 1, 1, 0, 1}	{0, 0, 0, 0, 0, -1, 1, 0}	{3, 2, 4, 3, 2, 1, 0, 2}
92	$x_1 \Phi_{o_1 \hat{L}}^-$	000011000 ₂	{0, 0, 0, 0, 1, 0, 1, 1}	{0, 0, 0, 0, 0, 0, -1, -1}	{-4, -3, -5, -4, -3, -2, -1, -2}
93	$g^x \bar{g}_{m_m \hat{L}}^-$	000110000 ₂	{0, 0, 0, 0, 0, 1, 1, 1}	{0, 0, 0, 0, 0, 0, -1, 1}	{-3, -2, -5, -4, -3, -2, -1, -2}

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
94	$e_\tau \frac{\cdot}{\text{L}}_{y_d}$	000000011 ₂	{1, 1, 1, 1, 0, 0, 0, 0}	{ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2},$ $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ }	{-2, -1, -2, -2, -2, -2, -1, -1}
95	$\overline{\nu_\mu} \frac{0}{\text{L}}_{w_m}$	011001010 ₂	{1, 1, 1, 0, 1, 0, 0, 0}	{ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2},$ $\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ }	{-2, -1, -2, -2, -2, -1, -1, -1}
96	$\overline{C} \frac{-\frac{2}{3}}{\text{r}_m \text{L}}$	011011010 ₂	{1, 1, 1, 0, 0, 1, 0, 0}	{ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2},$ $-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ }	{-2, -1, -2, -2, -2, -1, 0, -1}
97	$\overline{C} \frac{-\frac{2}{3}}{g_m \text{L}}$	011101010 ₂	{1, 1, 1, 0, 0, 0, 1, 0}	{ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2},$ $-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$ }	{1, 1, 2, 1, 0, 0, 0, 1}
98	$\overline{C} \frac{-\frac{2}{3}}{b_m \text{L}}$	011111010 ₂	{1, 1, 1, 0, 0, 0, 0, 1}	{ $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2},$ $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$ }	{-2, -1, -3, -3, -3, -2, -1, -1}
99	$\overline{\nu_\mu} \frac{0}{\text{R}}_{w_m}$	011001110 ₂	{1, 1, 0, 1, 1, 0, 0, 0}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2},$ $\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ }	{-2, -1, -2, -2, -1, -1, -1, -1}
100	$\overline{C} \frac{-\frac{2}{3}}{\text{r}_m \text{R}}$	011011110 ₂	{1, 1, 0, 1, 0, 1, 0, 0}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2},$ $-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ }	{-2, -1, -2, -2, -1, -1, 0, -1}
101	$\overline{C} \frac{-\frac{2}{3}}{g_m \text{R}}$	011101110 ₂	{1, 1, 0, 1, 0, 0, 1, 0}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2},$ $-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$ }	{1, 1, 2, 1, 1, 0, 0, 1}
102	$\overline{C} \frac{-\frac{2}{3}}{b_m \text{R}}$	011111110 ₂	{1, 1, 0, 1, 0, 0, 0, 1}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2},$ $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$ }	{-2, -1, -3, -3, -2, -2, -1, -1}
103	$\overline{b} \frac{\frac{1}{3}}{\text{o}_1 \text{R}}$	010010111 ₂	{1, 1, 0, 0, 1, 1, 0, 0}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2},$ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$ }	{-2, -1, -2, -2, -1, 0, 0, -1}
104	$\overline{b} \frac{\frac{1}{3}}{\text{c}_1 \text{R}}$	010100111 ₂	{1, 1, 0, 0, 1, 0, 1, 0}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2},$ $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$ }	{1, 1, 2, 1, 1, 1, 0, 1}
105	$\overline{b} \frac{\frac{1}{3}}{\text{m}_1 \text{R}}$	010110111 ₂	{1, 1, 0, 0, 1, 0, 0, 1}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2},$ $\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$ }	{-2, -1, -3, -3, -2, -1, -1, -1}
106	$t \frac{\frac{2}{3}}{\text{b}_d \text{L}}$	001110011 ₂	{1, 1, 0, 0, 0, 1, 1, 0}	{ $\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2},$ $-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}$ }	{1, 1, 2, 1, 1, 1, 1, 1}

107	$\overline{t}_{\hat{g}_d}^{\frac{2}{3}}$	001100011 ₂	{1, 1, 0, 0, 0, 1, 0, 1}	$\left\{ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-2, -1, -3, -3, -2, -1, 0, -1}
108	$\overline{t}_{\hat{r}_d}^{\frac{2}{3}}$	001010011 ₂	{1, 1, 0, 0, 0, 0, 1, 1}	$\left\{ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{1, 1, 1, 0, 0, 0, 0, 1}
109	$\overline{\nu_{\mu}}_{\hat{w}_d}^0$	011000010 ₂	{1, 0, 1, 1, 1, 0, 0, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{-1, -1, -1, -1, -1, -1, -1, -1}
110	$\overline{\overline{C}}_{\hat{r}_d}^{-\frac{2}{3}}$	011010010 ₂	{1, 0, 1, 1, 0, 1, 0, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{-1, -1, -1, -1, -1, -1, 0, -1}
111	$\overline{\overline{C}}_{\hat{g}_d}^{-\frac{2}{3}}$	011100010 ₂	{1, 0, 1, 1, 0, 0, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{2, 1, 3, 2, 1, 0, 0, 1}
112	$\overline{\overline{C}}_{\hat{b}_d}^{-\frac{2}{3}}$	011110010 ₂	{1, 0, 1, 1, 0, 0, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-1, -1, -2, -2, -2, -2, -1, -1}
113	$\overline{\overline{b}}_{\hat{o}_m}^{\frac{1}{3}}$	010011011 ₂	{1, 0, 1, 0, 1, 1, 0, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{-1, -1, -1, -1, -1, 0, 0, -1}
114	$\overline{\overline{b}}_{\hat{c}_m}^{\frac{1}{3}}$	010101011 ₂	{1, 0, 1, 0, 1, 0, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{2, 1, 3, 2, 1, 1, 0, 1}
115	$\overline{\overline{b}}_{\hat{m}_m}^{\frac{1}{3}}$	010111011 ₂	{1, 0, 1, 0, 1, 0, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{-1, -1, -2, -2, -2, -1, -1, -1}
116	$\overline{t}_{\hat{b}_m}^{\frac{2}{3}}$	001111111 ₂	{1, 0, 1, 0, 0, 1, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{2, 1, 3, 2, 1, 1, 1, 1}
117	$\overline{t}_{\hat{g}_m}^{\frac{2}{3}}$	001101111 ₂	{1, 0, 1, 0, 0, 1, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-1, -1, -2, -2, -2, -1, 0, -1}
118	$\overline{t}_{\hat{r}_m}^{\frac{2}{3}}$	001011111 ₂	{1, 0, 1, 0, 0, 0, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 1, 2, 1, 0, 0, 0, 1}
119	$\overline{\overline{t}}_{\hat{r}_m}^{-\frac{2}{3}}$	011011011 ₂	{1, 0, 0, 1, 1, 1, 0, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{-1, -1, -1, -1, 0, 0, 0, -1}
120	$\overline{\overline{t}}_{\hat{g}_m}^{-\frac{2}{3}}$	011101011 ₂	{1, 0, 0, 1, 1, 0, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{2, 1, 3, 2, 2, 1, 0, 1}

121	$\overline{t}^{\frac{2}{3}}$ $b_m^{\frac{-1}{3}}$	0111110112	{1, 0, 0, 1, 1, 0, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-1, -1, -2, -2, -1, -1, -1, -1}
122	$b^{\frac{1}{3}}$ $m_n^{\frac{-1}{3}}$	0001111112	{1, 0, 0, 1, 0, 1, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{2, 1, 3, 2, 2, 1, 1, 1}
123	$b^{\frac{1}{3}}$ $c_m^{\frac{-1}{3}}$	0001011112	{1, 0, 0, 1, 0, 1, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-1, -1, -2, -2, -1, -1, 0, -1}
124	$b^{\frac{1}{3}}$ $o_m^{\frac{-1}{3}}$	0000111112	{1, 0, 0, 1, 0, 0, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 1, 2, 1, 1, 0, 0, 1}
125	$c^{\frac{2}{3}}$ $b_1^{\frac{-1}{3}}$	0011101102	{1, 0, 0, 0, 1, 1, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{2, 1, 3, 2, 2, 2, 1, 1}
126	$c^{\frac{2}{3}}$ $g_1^{\frac{-1}{3}}$	0011001102	{1, 0, 0, 0, 1, 1, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-1, -1, -2, -2, -1, 0, 0, -1}
127	$c^{\frac{2}{3}}$ $r_1^{\frac{-1}{3}}$	0010101102	{1, 0, 0, 0, 1, 0, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 1, 2, 1, 1, 1, 0, 1}
128	ν_μ^0 $w_1^{\frac{-1}{3}}$	0010001102	{1, 0, 0, 0, 0, 1, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 1, 2, 1, 1, 1, 1, 1}
129	$\overline{\nu_\mu}^0$ $w_1^{\frac{-1}{3}}$	0110001102	{0, 1, 1, 1, 1, 0, 0, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-2, -1, -2, -1, -1, -1, -1, -1}
130	$\overline{C}^{\frac{2}{3}}$ $r_1^{\frac{-1}{3}}$	0110101102	{0, 1, 1, 1, 0, 1, 0, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	{-2, -1, -2, -1, -1, -1, 0, -1}
131	$\overline{C}^{\frac{2}{3}}$ $g_1^{\frac{-1}{3}}$	0111001102	{0, 1, 1, 1, 0, 0, 1, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{1, 1, 2, 2, 1, 0, 0, 1}
132	$\overline{C}^{\frac{2}{3}}$ $b_1^{\frac{-1}{3}}$	0111101102	{0, 1, 1, 1, 0, 0, 0, 1}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-2, -1, -3, -2, -2, -2, -1, -1}
133	$\overline{b}^{\frac{1}{3}}$ $o_m^{\frac{-1}{3}}$	0100111112	{0, 1, 1, 0, 1, 1, 0, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{-2, -1, -2, -1, -1, 0, 0, -1}
134	$\overline{b}^{\frac{1}{3}}$ $c_m^{\frac{-1}{3}}$	0101011112	{0, 1, 1, 0, 1, 0, 1, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{1, 1, 2, 2, 1, 1, 0, 1}

135	$\overline{b}_{\hat{m}}^{\frac{1}{3}}$	010111111 ₂	{0, 1, 1, 0, 1, 0, 0, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-2, -1, -3, -2, -2, -1, -1, -1}
136	$t_{\hat{b}_m}^{\frac{2}{3}}$	001111011 ₂	{0, 1, 1, 0, 0, 1, 1, 0}	$\left\{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{1, 1, 2, 2, 1, 1, 1, 1}
137	$t_{\hat{g}_m}^{\frac{2}{3}}$	001101011 ₂	{0, 1, 1, 0, 0, 1, 0, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-2, -1, -3, -2, -2, -1, 0, -1}
138	$t_{\hat{r}_m}^{\frac{2}{3}}$	001011011 ₂	{0, 1, 1, 0, 0, 0, 1, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	{1, 1, 1, 1, 0, 0, 0, 1}
139	$\overline{t}_{\hat{r}_m}^{-\frac{2}{3}}$	011011111 ₂	{0, 1, 0, 1, 1, 1, 0, 0}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-2, -1, -2, -1, 0, 0, 0, -1}
140	$\overline{t}_{\hat{g}_m}^{-\frac{2}{3}}$	011101111 ₂	{0, 1, 0, 1, 1, 0, 1, 0}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{1, 1, 2, 2, 2, 1, 0, 1}
141	$\overline{t}_{\hat{b}_m}^{-\frac{2}{3}}$	011111111 ₂	{0, 1, 0, 1, 1, 0, 0, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-2, -1, -3, -2, -1, -1, -1, -1}
142	$b_{\hat{m}}^{-\frac{1}{3}}$	000111011 ₂	{0, 1, 0, 1, 0, 1, 1, 0}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{1, 1, 2, 2, 2, 1, 1, 1}
143	$b_{\hat{c}_m}^{-\frac{1}{3}}$	000101011 ₂	{0, 1, 0, 1, 0, 1, 0, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-2, -1, -3, -2, -1, -1, 0, -1}
144	$b_{\hat{o}_m}^{-\frac{1}{3}}$	000011011 ₂	{0, 1, 0, 1, 0, 0, 1, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	{1, 1, 1, 1, 1, 0, 0, 1}
145	$c_{\hat{b}_d}^{\frac{2}{3}}$	001110010 ₂	{0, 1, 0, 0, 1, 1, 1, 0}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{1, 1, 2, 2, 2, 2, 1, 1}
146	$c_{\hat{g}_d}^{\frac{2}{3}}$	001100010 ₂	{0, 1, 0, 0, 1, 1, 0, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-2, -1, -3, -2, -1, 0, 0, -1}
147	$c_{\hat{r}_d}^{\frac{2}{3}}$	001010010 ₂	{0, 1, 0, 0, 1, 0, 1, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	{1, 1, 1, 1, 1, 1, 0, 1}
148	$\nu_{\mu}^{\hat{w}_d}{}^0$	001000010 ₂	{0, 1, 0, 0, 0, 1, 1, 1}	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	{1, 1, 1, 1, 1, 1, 1, 1}

149	$\overline{t}_{x_d}^{\frac{-2}{3}}$	011010011 ₂	{0, 0, 1, 1, 1, 1, 0, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{-1, -1, -1, 0, 0, 0, 0, -1}
150	$\overline{t}_{g_d}^{\frac{-2}{3}}$	011100011 ₂	{0, 0, 1, 1, 1, 0, 1, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{2, 1, 3, 3, 2, 1, 0, 1}
151	$\overline{t}_{b_d}^{\frac{-2}{3}}$	011110011 ₂	{0, 0, 1, 1, 1, 0, 0, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-1, -1, -2, -1, -1, -1, -1, -1}
152	$b_{m_1}^{\frac{1}{3}}$	000110111 ₂	{0, 0, 1, 1, 0, 1, 1, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\}$	{2, 1, 3, 3, 2, 1, 1, 1}
153	$b_{c_1}^{\frac{1}{3}}$	000100111 ₂	{0, 0, 1, 1, 0, 1, 0, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-1, -1, -2, -1, -1, -1, 0, -1}
154	$b_{o_1}^{\frac{1}{3}}$	000010111 ₂	{0, 0, 1, 1, 0, 0, 1, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{2, 1, 2, 2, 1, 0, 0, 1}
155	$c_{b_m}^{\frac{2}{3}}$	001111110 ₂	{0, 0, 1, 0, 1, 1, 1, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{2, 1, 3, 3, 2, 2, 1, 1}
156	$c_{g_m}^{\frac{2}{3}}$	001101110 ₂	{0, 0, 1, 0, 1, 1, 0, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-1, -1, -2, -1, -1, 0, 0, -1}
157	$c_{r_m}^{\frac{2}{3}}$	001011110 ₂	{0, 0, 1, 0, 1, 0, 1, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{2, 1, 2, 2, 1, 1, 0, 1}
158	$\nu_\mu w_m^0$	001001110 ₂	{0, 0, 1, 0, 0, 1, 1, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{2, 1, 2, 2, 1, 1, 1, 1}
159	$c_{b_m}^{\frac{2}{3}}$	001111010 ₂	{0, 0, 0, 1, 1, 1, 1, 0}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\}$	{2, 1, 3, 3, 3, 2, 1, 1}
160	$c_{g_m}^{\frac{2}{3}}$	001101010 ₂	{0, 0, 0, 1, 1, 1, 0, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{-1, -1, -2, -1, 0, 0, 0, -1}
161	$c_{r_m}^{\frac{2}{3}}$	001011010 ₂	{0, 0, 0, 1, 1, 0, 1, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{2, 1, 2, 2, 2, 1, 0, 1}
162	$\nu_\mu w_m^0$	001001010 ₂	{0, 0, 0, 1, 0, 1, 1, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{2, 1, 2, 2, 2, 1, 1, 1}
163	$\overline{e}_\tau y_d^+$	010000011 ₂	{0, 0, 0, 0, 1, 1, 1, 1}	$\left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right\}$	{2, 1, 2, 2, 2, 2, 1, 1}

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
164	$\overline{g^r g}^+$ $\overline{m_m} \overline{L}$	010110000 ₂	{1, 1, 1, 1, 1, 0, 0, 0}	{0, 0, 0, 0, 0, 0, 1, -1}	{3, 2, 5, 4, 3, 2, 1, 2}
165	$\overline{x_1 \Phi}^+$ $\overline{o_1} \overline{L}$	010011000 ₂	{1, 1, 1, 1, 0, 1, 0, 0}	{0, 0, 0, 0, 0, 0, 1, 1}	{4, 3, 5, 4, 3, 2, 1, 2}
166	$\overline{g^r b}^+$ $\overline{c_m} \overline{L}$	010100000 ₂	{1, 1, 1, 1, 0, 0, 1, 0}	{0, 0, 0, 0, 0, 1, -1, 0}	{-3, -2, -4, -3, -2, -1, 0, -2}
167	$\overline{g^a b}^+$ $\overline{o_m} \overline{L}$	010010000 ₂	{1, 1, 1, 1, 0, 0, 0, 1}	{0, 0, 0, 0, 0, 1, 0, -1}	{0, 0, 1, 1, 1, 1, 1, 0}
168	$\overline{x_2 \Phi}^+$ $\overline{c_1} \overline{L}$	010101000 ₂	{1, 1, 1, 0, 1, 1, 0, 0}	{0, 0, 0, 0, 0, 1, 0, 1}	{1, 1, 1, 1, 1, 1, 1, 0}
169	$\overline{x_3 \Phi}^+$ $\overline{m_1} \overline{L}$	010111000 ₂	{1, 1, 1, 0, 1, 0, 1, 0}	{0, 0, 0, 0, 0, 1, 1, 0}	{4, 3, 6, 5, 4, 3, 2, 2}
170	$\overline{x_1 \Phi}^+$ $\overline{o_d} \overline{R}$	010010100 ₂	{1, 1, 1, 0, 1, 0, 0, 1}	{0, 0, 0, 0, 1, -1, 0, 0}	{0, 0, 0, 0, 0, 0, -1, 0}
171	$\overline{x_2 \Phi}^+$ $\overline{c_d} \overline{R}$	010100100 ₂	{1, 1, 1, 0, 0, 1, 1, 0}	{0, 0, 0, 0, 1, 0, -1, 0}	{-3, -2, -4, -3, -2, -1, -1, -2}
172	$\overline{x_3 \Phi}^+$ $\overline{m_d} \overline{R}$	010110100 ₂	{1, 1, 1, 0, 0, 1, 0, 1}	{0, 0, 0, 0, 1, 0, 0, -1}	{0, 0, 1, 1, 1, 1, 0, 0}
173	$x_3 \Phi^-$ $\overline{m_m} \overline{R}$	000111100 ₂	{1, 1, 1, 0, 0, 0, 1, 1}	{0, 0, 0, 0, 1, 0, 0, 1}	{1, 1, 1, 1, 1, 1, 0, 0}
174	$x_2 \Phi^-$ $\overline{c_m} \overline{R}$	000101100 ₂	{1, 1, 0, 1, 1, 1, 0, 0}	{0, 0, 0, 0, 1, 0, 1, 0}	{4, 3, 6, 5, 4, 3, 1, 2}
175	$x_1 \Phi^-$ $\overline{o_m} \overline{R}$	000011100 ₂	{1, 1, 0, 1, 1, 0, 1, 0}	{0, 0, 0, 0, 1, 1, 0, 0}	{1, 1, 2, 2, 2, 2, 1, 0}
176	$\overline{v_e}^0$ $\overline{w_1} \overline{R}$	011000101 ₂	{1, 1, 0, 1, 1, 0, 0, 1}	{0, 0, 0, 1, -1, 0, 0, 0}	{0, 0, 0, 0, 0, -1, 0, 0}
177	$u^{\frac{2}{3}}$ $\overline{r_d} \overline{L}$	001010001 ₂	{1, 1, 0, 1, 0, 1, 1, 0}	{0, 0, 0, 1, 0, -1, 0, 0}	{0, 0, 0, 0, 0, -1, -1, 0}
178	$u^{\frac{2}{3}}$ $\overline{g_d} \overline{L}$	001100001 ₂	{1, 1, 0, 1, 0, 1, 0, 1}	{0, 0, 0, 1, 0, 0, -1, 0}	{-3, -2, -4, -3, -2, -2, -1, -2}
179	$u^{\frac{2}{3}}$ $\overline{b_d} \overline{L}$	001110001 ₂	{1, 1, 0, 1, 0, 0, 1, 1}	{0, 0, 0, 1, 0, 0, 0, -1}	{0, 0, 1, 1, 1, 0, 0, 0}
180	$\overline{d}^{\frac{1}{3}}$ $\overline{m_1} \overline{R}$	010110101 ₂	{1, 1, 0, 0, 1, 1, 1, 0}	{0, 0, 0, 1, 0, 0, 0, 1}	{1, 1, 1, 1, 1, 0, 0, 0}
181	$\overline{d}^{\frac{1}{3}}$ $\overline{c_1} \overline{R}$	010100101 ₂	{1, 1, 0, 0, 1, 1, 0, 1}	{0, 0, 0, 1, 0, 0, 1, 0}	{4, 3, 6, 5, 4, 2, 1, 2}

182	$\overline{d}_{\hat{o}_1 \overset{\vee}{R}}^{\frac{1}{3}}$	010010101 ₂	{1, 1, 0, 0, 1, 0, 1, 1}	{0, 0, 0, 1, 0, 1, 0, 0}	{1, 1, 2, 2, 2, 1, 1, 0}
183	$e_{\hat{y}_d \overset{\vee}{L}}^{-}$	000000001 ₂	{1, 1, 0, 0, 0, 1, 1, 1}	{0, 0, 0, 1, 1, 0, 0, 0}	{1, 1, 2, 2, 2, 1, 0, 0}
184	$\overline{B}_{\hat{b}_m \overset{\vee}{R}}^0$	011111100 ₂	{1, 0, 1, 1, 1, 1, 0, 0}	{0, 0, 1, -1, 0, 0, 0, 0}	{0, 0, 0, 0, -1, 0, 0, 0}
185	$\overline{v_e}_{\hat{w}_d \overset{\vee}{L}}^0$	011000001 ₂	{1, 0, 1, 1, 1, 0, 1, 0}	{0, 0, 1, 0, -1, 0, 0, 0}	{0, 0, 0, 0, -1, -1, 0, 0}
186	$u_{\hat{r}_1 \overset{\vee}{R}}^{\frac{2}{3}}$	001010101 ₂	{1, 0, 1, 1, 1, 0, 0, 1}	{0, 0, 1, 0, 0, -1, 0, 0}	{0, 0, 0, 0, -1, -1, -1, 0}
187	$u_{\hat{g}_1 \overset{\vee}{R}}^{\frac{2}{3}}$	001100101 ₂	{1, 0, 1, 1, 0, 1, 1, 0}	{0, 0, 1, 0, 0, 0, -1, 0}	{-3, -2, -4, -3, -3, -2, -1, -2}
188	$u_{\hat{b}_1 \overset{\vee}{R}}^{\frac{2}{3}}$	001110101 ₂	{1, 0, 1, 1, 0, 1, 0, 1}	{0, 0, 1, 0, 0, 0, 0, -1}	{0, 0, 1, 1, 0, 0, 0, 0}
189	$\overline{d}_{\hat{m}_d \overset{\vee}{L}}^{\frac{1}{3}}$	010110001 ₂	{1, 0, 1, 1, 0, 0, 1, 1}	{0, 0, 1, 0, 0, 0, 0, 1}	{1, 1, 1, 1, 0, 0, 0, 0}
190	$\overline{d}_{\hat{c}_d \overset{\vee}{L}}^{\frac{1}{3}}$	010100001 ₂	{1, 0, 1, 0, 1, 1, 1, 0}	{0, 0, 1, 0, 0, 0, 1, 0}	{4, 3, 6, 5, 3, 2, 1, 2}
191	$\overline{d}_{\hat{o}_d \overset{\vee}{L}}^{\frac{1}{3}}$	010010001 ₂	{1, 0, 1, 0, 1, 1, 0, 1}	{0, 0, 1, 0, 0, 1, 0, 0}	{1, 1, 2, 2, 1, 1, 1, 0}
192	$e_{\hat{y}_1 \overset{\vee}{R}}^{-}$	000000101 ₂	{1, 0, 1, 0, 1, 0, 1, 1}	{0, 0, 1, 0, 1, 0, 0, 0}	{1, 1, 2, 2, 1, 1, 0, 0}
193	$\omega_{\hat{r}_m \overset{\vee}{L}}^0$	001010000 ₂	{1, 0, 1, 0, 0, 1, 1, 1}	{0, 0, 1, 1, 0, 0, 0, 0}	{1, 1, 2, 2, 1, 0, 0, 0}
194	$e_T \phi_{\hat{g}_m \overset{\vee}{R}}^0$	001101100 ₂	{1, 0, 0, 1, 1, 1, 1, 0}	{0, 1, -1, 0, 0, 0, 0, 0}	{-1, 0, -1, -1, 0, 0, 0, 0}
195	$e_S \phi_{\hat{r}_1 \overset{\vee}{L}}^0$	001011000 ₂	{1, 0, 0, 1, 1, 1, 0, 1}	{0, 1, 0, -1, 0, 0, 0, 0}	{-1, 0, -1, -1, -1, 0, 0, 0}
196	$\overline{v_e}_{\hat{w}_m \overset{\vee}{R}}^0$	011001101 ₂	{1, 0, 0, 1, 1, 0, 1, 1}	{0, 1, 0, 0, -1, 0, 0, 0}	{-1, 0, -1, -1, -1, -1, 0, 0}
197	$u_{\hat{r}_m \overset{\vee}{L}}^{\frac{2}{3}}$	001011001 ₂	{1, 0, 0, 1, 0, 1, 1, 1}	{0, 1, 0, 0, 0, -1, 0, 0}	{-1, 0, -1, -1, -1, -1, -1, 0}
198	$u_{\hat{g}_m \overset{\vee}{L}}^{\frac{2}{3}}$	001101001 ₂	{1, 0, 0, 0, 1, 1, 1, 1}	{0, 1, 0, 0, 0, 0, -1, 0}	{-4, -2, -5, -4, -3, -2, -1, -2}

199	$\underline{u}_{\hat{L}}^{\frac{2}{3}}$	001111001 ₂	{0, 1, 1, 1, 1, 1, 0, 0}	{0, 1, 0, 0, 0, 0, -1}	{-1, 0, 0, 0, 0, 0, 0}
200	$\overline{d}_{\overset{\vee}{R}}^{\frac{1}{3}}$	010111101 ₂	{0, 1, 1, 1, 1, 0, 1, 0}	{0, 1, 0, 0, 0, 0, 1}	{0, 1, 0, 0, 0, 0, 0, 0}
201	$\overline{d}_{\overset{\vee}{R}}^{\frac{1}{3}}$	010101101 ₂	{0, 1, 1, 1, 1, 0, 0, 1}	{0, 1, 0, 0, 0, 0, 1, 0}	{3, 3, 5, 4, 3, 2, 1, 2}
202	$\overline{d}_{\overset{\vee}{R}}^{\frac{1}{3}}$	010011101 ₂	{0, 1, 1, 1, 0, 1, 1, 0}	{0, 1, 0, 0, 0, 1, 0, 0}	{0, 1, 1, 1, 1, 1, 1, 0}
203	$e_{Y_m} \hat{L}^{-}$	000001001 ₂	{0, 1, 1, 1, 0, 1, 0, 1}	{0, 1, 0, 0, 1, 0, 0, 0}	{0, 1, 1, 1, 1, 1, 0, 0}
204	$e_S \phi_{\hat{R}}^0$	001010100 ₂	{0, 1, 1, 1, 0, 0, 1, 1}	{0, 1, 0, 1, 0, 0, 0, 0}	{0, 1, 1, 1, 1, 0, 0, 0}
205	$e_T \phi_{g_1} \hat{L}^0$	001101000 ₂	{0, 1, 1, 0, 1, 1, 1, 0}	{0, 1, 1, 0, 0, 0, 0, 0}	{0, 1, 1, 1, 0, 0, 0, 0}
206	$\overline{w}_R \overset{\vee}{L}^0$	011100000 ₂	{0, 1, 1, 0, 1, 1, 0, 1}	{1, -1, 0, 0, 0, 0, 0}	{1, 0, 1, 0, 0, 0, 0, 0}
207	$e_T \phi_{g_d} \hat{R}^0$	001100100 ₂	{0, 1, 1, 0, 1, 0, 1, 1}	{1, 0, -1, 0, 0, 0, 0}	{0, 0, 0, -1, 0, 0, 0, 0}
208	$\overline{e_T \phi}_{b_1} \hat{L}^0$	011111000 ₂	{0, 1, 1, 0, 0, 1, 1, 1}	{1, 0, 0, -1, 0, 0, 0, 0}	{0, 0, 0, -1, -1, 0, 0, 0}
209	$\overline{e}_{Y_m} \overset{\vee}{R}^+$	010001101 ₂	{0, 1, 0, 1, 1, 1, 1, 0}	{1, 0, 0, 0, -1, 0, 0, 0}	{0, 0, 0, -1, -1, -1, 0, 0}
210	$d_{O_m} \hat{L}^{-\frac{1}{3}}$	000011001 ₂	{0, 1, 0, 1, 1, 1, 0, 1}	{1, 0, 0, 0, 0, -1, 0, 0}	{0, 0, 0, -1, -1, -1, -1, 0}
211	$d_{C_m} \hat{L}^{-\frac{1}{3}}$	000101001 ₂	{0, 1, 0, 1, 1, 0, 1, 1}	{1, 0, 0, 0, 0, 0, -1, 0}	{-3, -2, -4, -4, -3, -2, -1, -2}
212	$d_{m_m} \hat{L}^{-\frac{1}{3}}$	000111001 ₂	{0, 1, 0, 1, 0, 1, 1, 1}	{1, 0, 0, 0, 0, 0, 0, -1}	{0, 0, 1, 0, 0, 0, 0, 0}
213	$\overline{u}_{B_m} \overset{\vee}{R}^{-\frac{2}{3}}$	011111101 ₂	{0, 1, 0, 0, 1, 1, 1, 1}	{1, 0, 0, 0, 0, 0, 0, 1}	{1, 1, 1, 0, 0, 0, 0, 0}
214	$\overline{u}_{g_m} \overset{\vee}{R}^{-\frac{2}{3}}$	011101101 ₂	{0, 0, 1, 1, 1, 1, 1, 0}	{1, 0, 0, 0, 0, 0, 1, 0}	{4, 3, 6, 4, 3, 2, 1, 2}
215	$\overline{u}_{r_m} \overset{\vee}{R}^{-\frac{2}{3}}$	011011101 ₂	{0, 0, 1, 1, 1, 1, 0, 1}	{1, 0, 0, 0, 0, 1, 0, 0}	{1, 1, 2, 1, 1, 1, 1, 0}
216	$v_e_{W_m} \hat{L}^0$	001001001 ₂	{0, 0, 1, 1, 1, 0, 1, 1}	{1, 0, 0, 0, 1, 0, 0, 0}	{1, 1, 2, 1, 1, 1, 0, 0}

217	$e_s \phi \overset{0}{\underset{b_d}{\wedge}} \hat{R}$	001110100_2	$\{0, 0, 1, 1, 0, 1, 1\}$	$\{1, 0, 0, 1, 0, 0, 0\}$	$\{1, 1, 2, 1, 1, 0, 0\}$
218	$e_s \phi \overset{0}{\underset{r_m}{\wedge}} \hat{R}$	001011100_2	$\{0, 0, 1, 0, 1, 1, 1\}$	$\{1, 0, 1, 0, 0, 0, 0\}$	$\{1, 1, 2, 1, 0, 0, 0\}$
219	$w \overset{0}{\underset{b_m}{\wedge}} \hat{L}$	001110000_2	$\{0, 0, 0, 1, 1, 1, 1\}$	$\{1, 1, 0, 0, 0, 0, 0\}$	$\{0, 1, 1, 0, 0, 0, 0\}$

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
220	$\bar{t} \overset{-\frac{2}{3}}{\underset{r_1}{\wedge}} \hat{R}$	011010111_2	$\{1, 1, 1, 1, 1, 0, 0\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \right\}$	$\{-1, 0, 0, 0, 0, 0, -1\}$
221	$\bar{t} \overset{-\frac{2}{3}}{\underset{g_1}{\wedge}} \hat{R}$	011100111_2	$\{1, 1, 1, 1, 1, 0, 0\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	$\{2, 2, 4, 3, 2, 1, 0, 1\}$
222	$\bar{t} \overset{-\frac{2}{3}}{\underset{b_1}{\wedge}} \hat{R}$	011110111_2	$\{1, 1, 1, 1, 1, 0, 0\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	$\{-1, 0, -1, -1, -1, -1, -1, -1\}$
223	$b \overset{\frac{1}{3}}{\underset{m_d}{\wedge}} \hat{L}$	000110011_2	$\{1, 1, 1, 1, 0, 1, 0\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	$\{2, 2, 4, 3, 2, 1, 1, 1\}$
224	$b \overset{\frac{1}{3}}{\underset{c_d}{\wedge}} \hat{L}$	000100011_2	$\{1, 1, 1, 1, 0, 1, 0\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	$\{-1, 0, -1, -1, -1, -1, 0, -1\}$
225	$b \overset{\frac{1}{3}}{\underset{o_d}{\wedge}} \hat{L}$	000010011_2	$\{1, 1, 1, 1, 0, 0, 1, 1\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	$\{2, 2, 3, 2, 1, 0, 0, 1\}$
226	$s \overset{\frac{1}{3}}{\underset{m_m}{\wedge}} \hat{R}$	000111110_2	$\{1, 1, 1, 0, 1, 1, 1, 0\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	$\{2, 2, 4, 3, 2, 2, 1, 1\}$
227	$s \overset{\frac{1}{3}}{\underset{c_m}{\wedge}} \hat{R}$	000101110_2	$\{1, 1, 1, 0, 1, 1, 0, 1\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	$\{-1, 0, -1, -1, -1, 0, 0, -1\}$
228	$s \overset{\frac{1}{3}}{\underset{o_m}{\wedge}} \hat{R}$	000011110_2	$\{1, 1, 1, 0, 1, 0, 1, 1\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	$\{2, 2, 3, 2, 1, 1, 0, 1\}$
229	$e_\mu \overset{-}{\underset{y_m}{\wedge}} \hat{R}$	000001110_2	$\{1, 1, 1, 0, 0, 1, 1, 1\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	$\{2, 2, 3, 2, 1, 1, 1, 1\}$
230	$s \overset{\frac{1}{3}}{\underset{m_m}{\wedge}} \hat{L}$	000111010_2	$\{1, 1, 0, 1, 1, 1, 0, 0\}$	$\left\{ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	$\{2, 2, 4, 3, 3, 2, 1, 1\}$

231	$\mathbf{s}_{\mathbf{c}_m}^{\frac{1}{-\frac{1}{3}}}$	000101010 ₂	{1, 1, 0, 1, 1, 1, 0, 1}	$\left\{ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-1, 0, -1, -1, 0, 0, 0, -1}
232	$\mathbf{s}_{\mathbf{o}_m}^{\frac{1}{-\frac{1}{3}}}$	000011010 ₂	{1, 1, 0, 1, 1, 0, 1, 1}	$\left\{ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 2, 3, 2, 2, 1, 0, 1}
233	$\mathbf{e}_{\mu}^{\frac{-}{\hat{\mathbf{L}}}}_{\mathbf{Y}_m}$	000001010 ₂	{1, 1, 0, 1, 0, 1, 1, 1}	$\left\{ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 2, 3, 2, 2, 1, 1, 1}
234	$\overline{\mathbf{e}_{\tau}}^{\frac{+}{\hat{\mathbf{R}}}}_{\mathbf{Y}_1}$	010000111 ₂	{1, 1, 0, 0, 1, 1, 1, 1}	$\left\{ \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 2, 3, 2, 2, 2, 1, 1}
235	$\mathbf{s}_{\mathbf{m}_1}^{\frac{1}{-\frac{1}{3}}}$	000110110 ₂	{1, 0, 1, 1, 1, 1, 1, 0}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{3, 2, 5, 4, 3, 2, 1, 1}
236	$\mathbf{s}_{\mathbf{c}_1}^{\frac{1}{-\frac{1}{3}}}$	000100110 ₂	{1, 0, 1, 1, 1, 1, 0, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{0, 0, 0, 0, 0, 0, 0, -1}
237	$\mathbf{s}_{\mathbf{o}_1}^{\frac{1}{-\frac{1}{3}}}$	000010110 ₂	{1, 0, 1, 1, 1, 0, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{3, 2, 4, 3, 2, 1, 0, 1}
238	$\mathbf{e}_{\mu}^{\frac{-}{\hat{\mathbf{R}}}}_{\mathbf{Y}_1}$	000000110 ₂	{1, 0, 1, 1, 0, 1, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{3, 2, 4, 3, 2, 1, 1, 1}
239	$\overline{\mathbf{e}_{\tau}}^{\frac{+}{\hat{\mathbf{L}}}}_{\mathbf{Y}_m}$	010001011 ₂	{1, 0, 1, 0, 1, 1, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{3, 2, 4, 3, 2, 2, 1, 1}
240	$\overline{\mathbf{v}_{\tau}}^{\frac{0}{\hat{\mathbf{L}}}}_{\mathbf{w}_m}$	011001011 ₂	{1, 0, 0, 1, 1, 1, 1, 1}	$\left\{ \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{3, 2, 4, 3, 3, 2, 1, 1}
241	$\mathbf{s}_{\mathbf{m}_d}^{\frac{1}{-\frac{1}{3}}}$	000110010 ₂	{0, 1, 1, 1, 1, 1, 1, 0}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right\}$	{2, 2, 4, 4, 3, 2, 1, 1}
242	$\mathbf{s}_{\mathbf{c}_d}^{\frac{1}{-\frac{1}{3}}}$	000100010 ₂	{0, 1, 1, 1, 1, 1, 0, 1}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{-1, 0, -1, 0, 0, 0, -1}
243	$\mathbf{s}_{\mathbf{o}_d}^{\frac{1}{-\frac{1}{3}}}$	000010010 ₂	{0, 1, 1, 1, 1, 0, 1, 1}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2} \right\}$	{2, 2, 3, 3, 2, 1, 0, 1}
244	$\mathbf{e}_{\mu}^{\frac{-}{\hat{\mathbf{L}}}}_{\mathbf{Y}_d}$	000000010 ₂	{0, 1, 1, 1, 0, 1, 1, 1}	$\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right\}$	{2, 2, 3, 3, 2, 1, 1, 1}

245	$\overline{e}_\tau \overset{+}{R}$	010001111_2	$\{0, 1, 1, 0, 1, 1, 1, 1\}$	$\left\{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	$\{2, 2, 3, 3, 2, 2, 1, 1\}$
246	$\overline{v}_\tau \overset{0}{R}$	011001111_2	$\{0, 1, 0, 1, 1, 1, 1, 1\}$	$\left\{-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	$\{2, 2, 3, 3, 3, 2, 1, 1\}$
247	$\overline{v}_\tau \overset{0}{L}$	011000011_2	$\{0, 0, 1, 1, 1, 1, 1, 1\}$	$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	$\{3, 2, 4, 4, 3, 2, 1, 1\}$

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
248	$\text{Ex2} \overset{0}{R}$	001001100_2	$\{1, 1, 1, 1, 1, 1, 0\}$	$\{1, 0, 0, 0, 0, 0, 0, 0\}$	$\left\{\frac{1}{2}, \frac{1}{2}, 1, 0, 0, 0, 0, 0\right\}$
249	$\text{Ex2} \overset{0}{L}$	001001000_2	$\{1, 1, 1, 1, 1, 0, 1\}$	$\{0, 1, 0, 0, 0, 0, 0, 0\}$	$\left\{-\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0\right\}$
250	$\text{Ex2} \overset{0}{R}$	001000100_2	$\{1, 1, 1, 1, 1, 0, 1\}$	$\{0, 0, 1, 0, 0, 0, 0, 0\}$	$\left\{\frac{1}{2}, \frac{1}{2}, 1, 1, 0, 0, 0, 0\right\}$
251	$\text{Ex2} \overset{0}{L}$	001000000_2	$\{1, 1, 1, 1, 0, 1, 1\}$	$\{0, 0, 0, 1, 0, 0, 0, 0\}$	$\left\{\frac{1}{2}, \frac{1}{2}, 1, 1, 0, 0, 0, 0\right\}$
252	$\text{Ex1} \overset{-}{R}$	000001100_2	$\{1, 1, 1, 0, 1, 1, 1\}$	$\{0, 0, 0, 0, 0, 1, 0, 0\}$	$\left\{\frac{1}{2}, \frac{1}{2}, 1, 1, 1, 0, 0, 0\right\}$
253	$\text{Ex1} \overset{-}{L}$	000001000_2	$\{1, 1, 0, 1, 1, 1, 1\}$	$\{0, 0, 0, 0, 0, 0, 1, 0\}$	$\left\{\frac{1}{2}, \frac{1}{2}, 1, 1, 1, 1, 0, 0\right\}$
254	$\text{Ex1} \overset{-}{R}$	000000100_2	$\{1, 0, 1, 1, 1, 1, 1\}$	$\{0, 0, 0, 0, 0, 0, 1, 0\}$	$\left\{\frac{7}{2}, \frac{5}{2}, 5, 4, 3, 2, 1, 2\right\}$
255	$\text{Ex1} \overset{-}{L}$	000000000_2	$\{0, 1, 1, 1, 1, 1, 1\}$	$\{0, 0, 0, 0, 0, 0, 0, 1\}$	$\left\{\frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0\right\}$

Seq #	Symbol	0apccssgg-Bits	Binary Coordinates	E8 Coordinates	Algebra Root
256	$\overline{v}_\tau \overset{0}{R}$	011000111_2	$\{1, 1, 1, 1, 1, 1, 1\}$	$\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}$	$\{3, 3, 5, 4, 3, 2, 1, 1\}$

Appendix C: User Interfaces of VisibLie_Dynkin_&_E8

E8 Parent

Reset Visualize Dynkin Type Finite Affine Hyper Very (19, 1) Parent

Project Algebra E8 to Coxeter plane Symmetry order E8(30) with Permutation: 1

Dimension=cmA Rank=8 DetCM=1 # of Positive Roots=120 Coxeter#=30 Group=> Hasse Coxeter Perm2D Visualizations Perm3D

CartanMatrix Root # Weights Positive Root Vectors Heights

2 -1 0 0 0 0 0 0	1	2 -1 0 0 0 0 0 0	1 0 0 0 0 0 0 0	1
-1 2 -1 0 0 0 0 0	2	-1 2 -1 0 0 0 0 0	0 1 0 0 0 0 0 0	1
0 -1 2 -1 0 0 0 0	3	0 -1 2 -1 0 0 0 0	0 0 1 0 0 0 0 0	1
0 0 -1 2 -1 0 0 0	4	0 0 -1 2 -1 0 0 0	0 0 0 1 0 0 0 0	1
0 0 0 -1 2 -1 0 -1	5	0 0 0 -1 2 -1 0 -1	0 0 0 0 1 0 0 0	1
0 0 0 0 -1 2 -1 0	6	0 0 0 0 -1 2 -1 0	0 0 0 0 0 1 0 0	1
0 0 0 0 0 -1 2 -1 0	7	0 0 0 0 0 -1 2 -1 0	0 0 0 0 0 0 1 0	1
0 0 0 0 0 0 -1 2 0	8	0 0 0 0 0 0 -1 0 2	0 0 0 0 0 0 0 1	1
0 0 0 0 0 0 -1 0 2	9	1 1 -1 0 0 0 0 0	1 1 0 0 0 0 0 0	2
	10	-1 1 1 -1 0 0 0 0	0 1 1 0 0 0 0 0	2
	11	0 -1 1 1 -1 0 0 0	0 0 1 1 0 0 0 0	2
	12	0 0 -1 1 1 1 -1 0	0 0 0 1 1 0 0 0	2
	13	0 0 0 -1 1 1 1 -1	0 0 0 0 1 1 0 0	2
	14	0 0 0 -1 1 1 -1 0 1	0 0 0 0 1 0 0 1	2
	15	0 0 0 0 -1 1 1 0	0 0 0 0 0 1 1 0	2
	16	1 0 1 -1 0 0 0 0	1 1 1 0 0 0 0 0	3
	17	-1 1 0 1 -1 0 0 0	0 1 1 1 0 0 0 0	3
	18	0 -1 1 0 1 -1 0 -1	0 0 1 1 1 0 0 0	3
	19	0 0 -1 1 0 1 -1 -1	0 0 0 1 1 1 0 0	3
	20	0 0 0 -1 1 0 1 -1	0 0 0 0 1 1 1 0	3
	21	0 0 -1 1 0 -1 0 1	0 0 0 1 1 0 0 1	3
	22	0 0 0 -1 0 1 -1 1	0 0 0 0 1 1 0 1	3
	23	0 0 -1 1 -1 1 -1 1	0 0 0 1 1 1 0 1	4
	24	1 0 0 1 -1 0 0 0	1 1 1 1 0 0 0 0	4
	25	-1 1 0 0 1 -1 0 -1	0 1 1 1 1 1 0 0	4
	26	0 -1 1 0 0 1 -1 -1	0 0 1 1 1 1 0 0	4
	27	0 -1 1 0 0 -1 0 1	0 0 1 1 1 0 0 1	4
	28	0 0 -1 1 0 0 1 -1	0 0 0 1 1 1 1 0	4
	29	0 0 0 -1 0 0 1 1	0 0 0 1 1 1 1 1	4
	30	0 -1 1 0 0 -1 1 -1	0 0 1 1 1 1 0 1	5
	31	1 0 0 0 1 -1 0 -1	1 1 1 1 1 1 0 0	5
	32	-1 1 0 0 0 1 -1 -1	0 1 1 1 1 1 0 0	5
	33	-1 1 0 0 0 -1 0 1	0 1 1 1 1 1 0 0 1	5
	34	0 0 -1 0 1 0 -1 0	0 0 0 1 2 1 0 0 1	5
	35	0 -1 1 0 0 0 1 -1	0 0 1 1 1 1 1 0	5
	36	0 0 -1 1 -1 0 1 1	0 0 0 1 1 1 1 1 1	5
	37	0 -1 1 0 -1 0 1 1	0 0 1 1 1 1 1 1 1	6
	38	1 0 0 0 0 1 -1 -1	1 1 1 1 1 1 1 0 0	6
	39	1 0 0 0 0 -1 0 1	1 1 1 1 1 1 0 0 1	6
	40	-1 1 0 0 0 0 1 -1	0 1 1 1 1 1 1 1 0	6
	41	-1 1 0 0 -1 1 -1 1	0 1 1 1 1 1 1 0 1	6
	42	0 -1 1 -1 1 0 -1 0	0 0 1 1 2 1 0 0 1	6
	43	0 0 -1 0 1 -1 1 0	0 0 0 1 2 1 1 1 1	6
	44	-1 1 0 0 -1 0 1 1	0 1 1 1 1 1 1 1 1	7
	45	1 0 0 0 0 0 1 -1	1 1 1 1 1 1 1 1 0	7

46	0 -1 1 -1 1 -1 1 0	0 0 1 1 2 1 1 1	7
47	0 -1 0 1 0 0 -1 0	0 0 1 2 2 1 0 1	7
48	1 0 0 0 -1 1 -1 1	1 1 1 1 1 1 0 1	7
49	-1 1 0 -1 1 0 -1 0	0 1 1 1 2 1 0 1	7
50	0 0 -1 0 0 1 0 0	0 0 0 1 2 2 1 1	7
51	1 0 0 -1 1 0 -1 0	1 1 1 1 2 1 0 1	8
52	-1 1 -1 1 0 -1 0	0 1 1 2 2 1 0 1	8
53	1 0 0 0 -1 0 1 1	1 1 1 1 1 1 1 1	8
54	-1 1 0 -1 1 -1 1 0	0 1 1 1 2 1 1 1	8
55	0 -1 1 -1 0 1 0 0	0 0 1 1 2 2 1 1	8
56	0 -1 0 1 0 -1 1 0	0 0 1 2 2 1 1 1	8
57	1 0 -1 1 0 0 -1 0	1 1 1 2 2 1 0 1	9
58	-1 0 1 0 0 0 -1 0	0 1 2 2 2 1 0 1	9
59	0 -1 0 1 -1 1 0 0	0 0 1 2 2 2 1 1	9
60	-1 1 -1 1 0 -1 1 0	0 1 1 2 2 1 1 1	9
61	1 0 0 -1 1 -1 1 0	1 1 1 1 2 1 1 1	9
62	-1 1 0 -1 0 1 0 0	0 1 1 1 2 2 1 1	9
63	1 0 0 -1 0 1 0 0	1 1 1 1 2 2 1 1	10
64	1 -1 1 0 0 0 -1 0	1 1 2 2 2 1 0 1	10
65	1 0 -1 1 0 -1 1 0	1 1 1 2 2 1 1 1	10
66	-1 1 -1 1 -1 1 0 0	0 1 1 2 2 2 1 1	10
67	-1 0 1 0 0 -1 1 0	0 1 2 2 2 1 1 1	10
68	0 -1 0 0 1 0 0 -1	0 0 1 2 3 2 1 1	10
69	0 1 0 0 0 0 -1 0	1 2 2 2 2 1 0 1	11
70	-1 1 -1 0 1 0 0 -1	0 1 1 2 3 2 1 1	11
71	1 -1 1 0 0 -1 1 0	1 1 2 2 2 1 1 1	11
72	-1 0 1 0 -1 1 0 0	0 1 2 2 2 2 1 1	11
73	1 0 -1 1 -1 1 0 0	1 1 1 2 2 2 1 1	11
74	0 -1 0 0 0 0 0 1	0 0 1 2 3 2 1 2	11
75	1 -1 1 0 -1 1 0 0	1 1 2 2 2 2 1 1	12
76	1 0 -1 0 1 0 0 -1	1 1 1 2 3 2 1 1	12
77	0 1 0 0 0 -1 1 0	1 2 2 2 2 1 1 1	12
78	-1 0 1 -1 1 0 0 -1	0 1 2 2 3 2 1 1	12
79	-1 1 -1 0 0 0 0 1	0 1 1 2 3 2 1 2	12
80	-1 0 0 1 0 0 0 -1	0 1 2 3 3 2 1 1	13
81	0 1 0 0 -1 1 0 0	1 2 2 2 2 2 1 1	13
82	-1 0 1 -1 0 0 0 1	0 1 2 2 3 2 1 2	13
83	1 -1 1 -1 1 0 0 -1	1 1 2 2 3 2 1 1	13
84	1 0 -1 0 0 0 0 1	1 1 1 2 3 2 1 2	13
85	-1 0 0 1 -1 0 0 1	0 1 2 3 3 2 1 2	14
86	1 -1 0 1 0 0 0 -1	1 1 2 3 3 2 1 1	14
87	0 1 0 -1 1 0 0 -1	1 2 2 2 3 2 1 1	14
88	1 -1 1 -1 0 0 0 1	1 1 2 2 3 2 1 2	14
89	0 1 -1 1 0 0 0 -1	1 2 2 3 3 2 1 1	15
90	1 -1 0 1 -1 0 0 1	1 1 2 3 3 2 1 2	15
91	0 1 0 -1 0 0 0 1	1 2 2 2 3 2 1 2	15
92	-1 0 0 0 1 -1 0 0	0 1 2 3 4 2 1 2	15
93	0 0 1 0 0 0 0 -1	1 2 3 3 3 2 1 1	16
94	1 -1 0 0 0 1 -1 0	1 1 2 3 4 2 1 2	16
95	0 1 -1 1 -1 0 0 1	1 2 2 3 3 2 1 2	16
96	-1 0 0 0 0 1 -1 0	0 1 2 3 4 3 1 2	16
97	0 0 1 0 -1 0 0 1	1 2 3 3 3 2 1 2	17
98	0 1 -1 0 1 -1 0 0	1 2 2 3 4 2 1 2	17
99	1 -1 0 0 0 1 -1 0	1 1 2 3 4 3 1 2	17
100	-1 0 0 0 0 1 0 1	0 1 2 3 4 3 2 2	17
101	0 1 -1 0 0 1 -1 0	1 2 2 3 4 3 1 2	18
102	0 0 1 -1 1 -1 0 0	1 2 3 3 4 2 1 2	18
103	1 -1 0 0 0 0 1 0	1 1 2 3 4 3 2 2	18
104	0 0 0 1 0 -1 0 0	1 2 3 4 4 2 1 2	19
105	0 0 1 -1 0 1 -1 0	1 2 3 3 4 3 1 2	19
106	0 1 -1 0 0 0 1 0	1 2 2 3 4 3 2 2	19
107	0 0 1 -1 0 0 1 0	1 2 3 3 4 3 2 2	20
108	0 0 0 1 -1 1 -1 0	1 2 3 4 4 3 1 2	20
109	0 0 0 1 -1 0 1 0	1 2 3 4 4 3 2 2	21
110	0 0 0 0 1 0 -1 -1	1 2 3 4 5 3 1 2	21
111	0 0 0 0 0 0 -1 1	1 2 3 4 5 3 1 3	22
112	0 0 0 0 0 1 -1 1 -1	1 2 3 4 5 3 2 2	22
113	0 0 0 0 0 1 0 -1	1 2 3 4 5 4 2 2	23
114	0 0 0 0 0 -1 1 1	1 2 3 4 5 3 2 3	23
115	0 0 0 0 -1 1 0 1	1 2 3 4 5 4 2 3	24
116	0 0 0 -1 1 0 0 0	1 2 3 4 6 4 2 3	25
117	0 0 -1 1 0 0 0 0	1 2 3 5 6 4 2 3	26
118	0 -1 1 0 0 0 0 0	1 2 4 5 6 4 2 3	27
119	-1 1 0 0 0 0 0 0	1 3 4 5 6 4 2 3	28
120	1 0 0 0 0 0 0 0	2 3 4 5 6 4 2 3	28

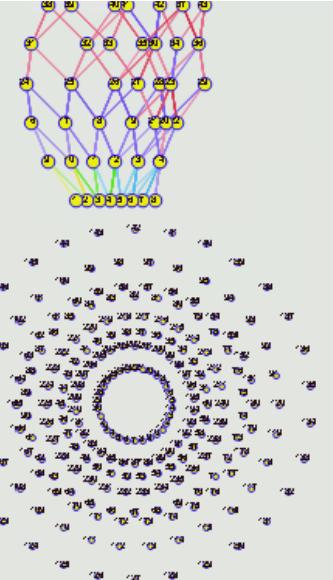
perm2D
perm3D

Figure 20: The VisibLie_Dynkin user interface, showing the roots, weights, heights with Coxeter diagrams

Refresh MetaFavorites E8 ↴OR ↴AND Data Sets: e8 ↴2 ArtPrint

inFile OutToFile FileOnly Dir Name "E8out31" .png ↴Particles=240 tLines=0

pSize nrml ↴ Zoom Exp ↴ 0 pScale ↴ 0.02 Ticks ↴ 4 Frame ↴ 1.5 Limit

Frame Center: $(x,y)=(0,0)$ ViewPoint $(x,y)=(1.3, 2.4)$




nDim CameraLoc ↴ 1 Rectify Edges ↴ 0 Edge Dimension ↴ 8

$z = 0$ $z = 2.$ pColors ↴ 0 oldColors

Show: Axes pVertices Edges ClickVerts Polygons Perspective pOverlaps pLabels pLocations pMassLife

2D 3D Stereo Anaglyph PhysEdge-NotCnts Clear ClickVerts 2D Face Select: (H, V, Z) (Z, H, V) (V, Z, H)

Norm'd Edges{Value,Count}: $\{\sqrt{2}, 6720\}$ ↴ eFrames ↴ 1+1 eWindow ↴ 1500x100

EdgeListAnim8 InnerFilter% ↴ 0 eColorPos eColors ↴ 10 BrightBands

Scale Surface: ↴ 24 Show: Surface Color Trialities T161 T161p T162 T162p T232 T232p eRadius ↴ 0.01

Binary (bitwise) Filter Type: OR AND pLists E8!=Excluded ↴ 2

SM Row(shape) 1=Leptons 2=Quarks 3=WeakStrong= $\omega-W/g$ 4=Higgs= $e\phi-B/x^2$ 5=Excluded=Dimensions

Anti(shape) 0 1 0=p(lepton=tri/quark=squ/boson=cir), 1=p(utr/dia/inv)

Type(color) 0 1 0=e/Ex14(y)/d-s-b/g/x $\delta(r/g/b)$, 1=v/Ex58(w)/u-c-t/ $\omega-W/e\phi-B(o/c/m)$

Color(color) 0 1 2 3 0=(y/w), 1=(o/r), 2=(c/g), 3=(m/b)

Spin(shade) 0 1 2 3 0= \hat{L} , 1= \hat{R} , 2= \hat{L} , 3= \hat{R} (light/med/dark)

Gen(size) 0 1 2 3 0=bosons(pSize), 1=e(tiny), 2= μ (nrml), 3= τ (huge)

t Steps ↴ 1 Trans Rot Trans&Rot Spin3D Identity PtCoords Favorites 15 ↴ dimLocs

WS	$\bar{W}\Gamma$	U	V	\bar{W}	r	g	b
H { 0.}	-0.55679; 34404; 52	0.196949; 25177	-0.196949; 92517; 7	0.080547; 726394; 4	-0.38529; 08761; 71	0.	0.385290; 876171
V { 0.180913; 155536	0.	0.160212; 955043	0.160212; 955043	0.	0.099017; 051654; 5	0.766360; 424875	0.099017; 051654; 5
WS	$\bar{W}\Gamma$	U	V	\bar{W}	r	g	b

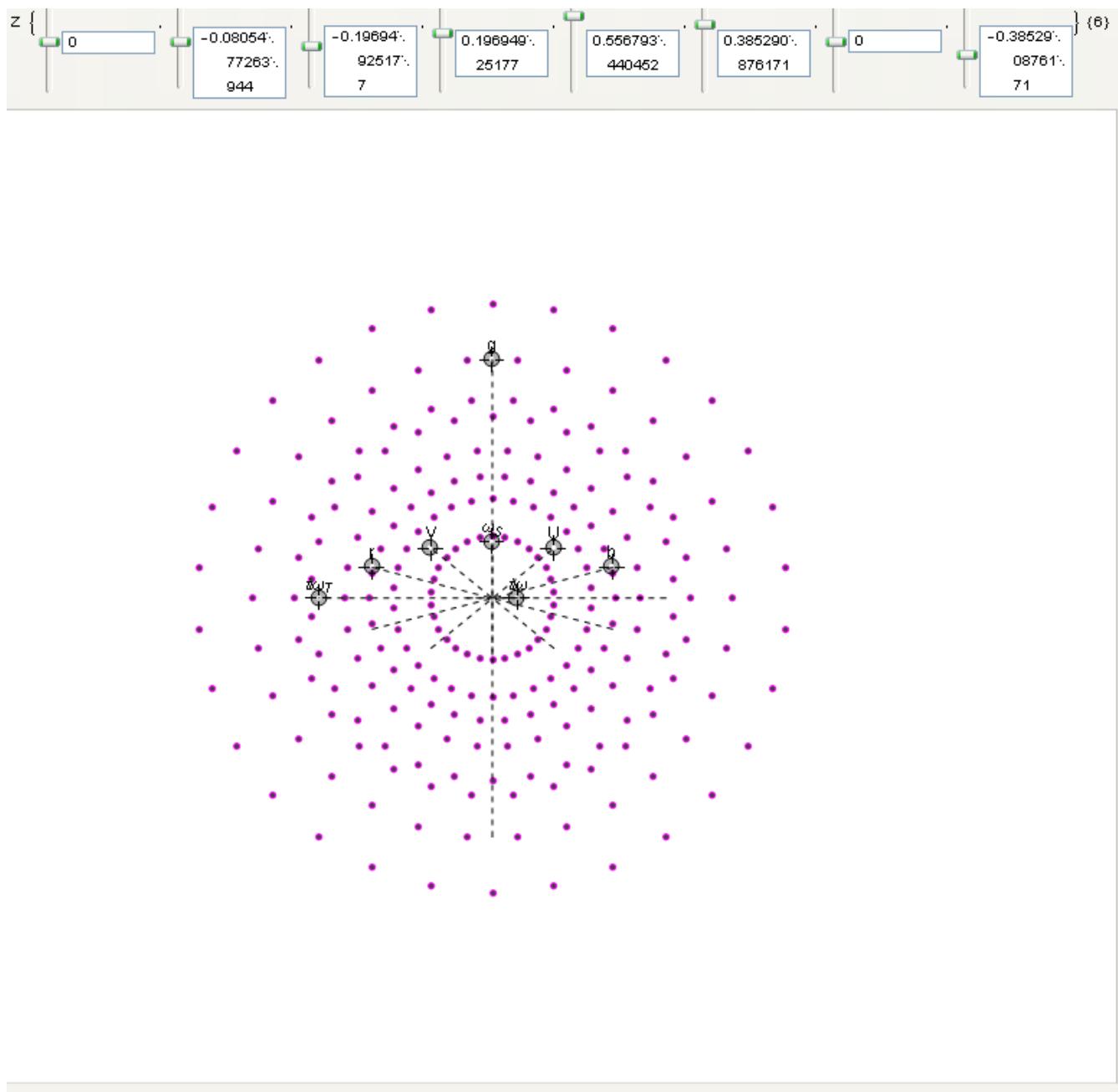


Figure 21: The VisibLie_E8 user interface

Appendix D: Exports to 3D Virtual Worlds

```
Import@"Snapshot_005.png"
```

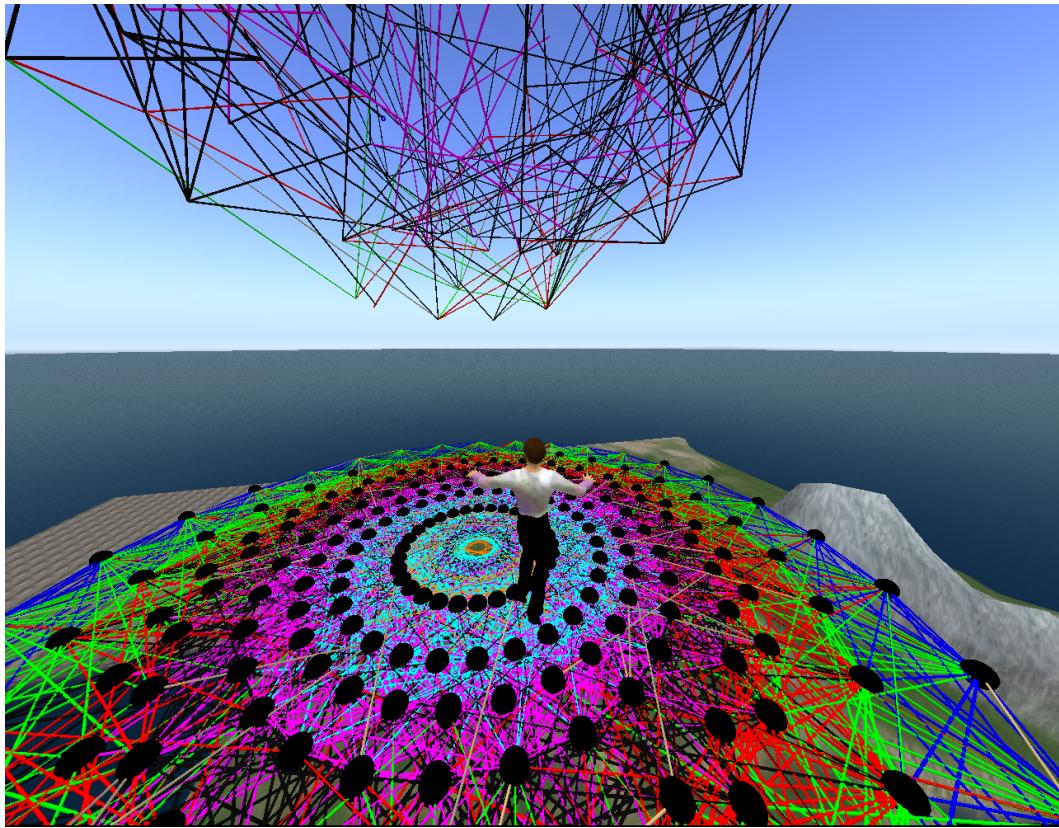


Figure 22: Virtual 2D photo of two E_8 region sized 3D objects exported from *Mathematica*TM and then "bot rez'd" into the OpenSimulator 3D Virtual World.

In addition to my avatar, this virtual world photo contains both the 8D \rightarrow 3D orthographic projection and its 2D orthographic shadow (the 8D \rightarrow 2D Petrie projection) of E_8 , with
13920 prims=2*(240 vertices + 6720 edges)